Threat, Commitment, and Brinkmanship in Adversarial Bargaining

Pablo Cuellar, Lucas Rentchler

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Motivation

- **Adversarial bargaining:**

  A party demands payment from another under threat of a welfare-destroying action

- Examples:
  - International conflict: demand of territory under threat of invasion
  - Plea bargaining: prosecutor threatens to take case to trial
  - Labor negotiation: better conditions or strike

- **Questions:**

1. Is the threat credible?
2. If it is credible, will the scale of the action be large enough?
3. Role of commitment power?

Preview of the results

- To make a credible threat, proposer chooses a low scale of the conflict
- A large-scale conflict is a non-credible threat, but it is optimal
  - Brinkmanship
- High commitment power (to not reduce the scale) can be detrimental
  - Commitment trap: it is not sequentially rational to keep a low scale

Contributions

- Characterization of credibility of the threat,
    - and election of the scale of the conflict
- Formalization of bargaining ideas from Thomas Schelling
    - Credible threats, commitment, brinkmanship
- Commitment Trap:
    - Commitment power to not reducing the scale of the action can be detrimental

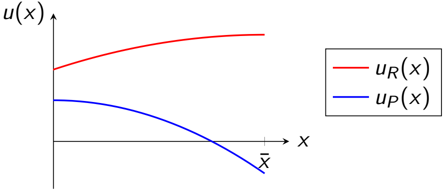
Existing work

- Pre-commitment: mostly to a demand
  - Schelling (1956, 1960, 1966), Crawford (1982), Muthoo (1992, 1996), Dutta (2012, 2021)
- Reputational bargaining: behavioral type
  - Abreu and Gul (2000), Kambe (1999), Abreu and Sethi (2003), Wolitzky (2012), Atakan and Ekmekci (2014), Sanktjohanser (2020), Ekmekci and Zhang (2021)
- Applications
  - Schwarz and Sonin (2008)

Model

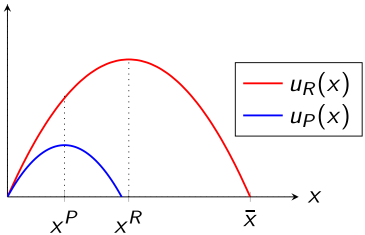
Model

- Proposer (P) demands a payment from the Responder (R)
- 2 stages:
  - Negotiation (multiperiod)
  - Resolution (conflict)
- Welfare-destroying conflict:
  - Payoffs are given by the P's *scale* $x \in [0, \bar{x}]$:
  - P's payoff: $u_P(x)$
  - R's loss: $u_R(x)$



Model (2)

- Payoff specification of the conflict is without loss of generality
- Example:



Model (3)

- Negotiation Stage:
  - Multiperiod: $t \in \mathbb{N} \equiv \{1, 2, ...\}$
  - Discount factor δ

- At each t:
  - P chooses a *intended scale* $x_I^t \in [0, \bar{x}]$
  - P offers a deal y to R
  - If accepted: $u_P = y$ and $u_R = -y$
  - If rejected: P decides whether to start the resolution stage

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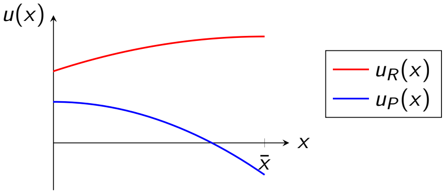
- Scaling-down costs: $k \cdot c(x_I^t, x_I^{t-1})$
  - Increasing in the magnitude of the reduction: $(x_I^{t-1} - x_I^t)$
  - $c(x_I^t, x_I^{t-1}) = 0$ if $x_I^t \geq x_I^{t-1}$
  - k: pre-commitment power

- If P decides no resolution stage: resolution stage starts with prob. p

Model (4)

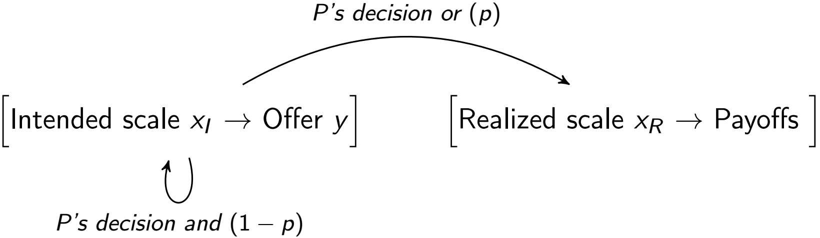
- Resolution Stage:
  - One-shot final stage
  - P chooses *realized scale* x_F, at a cost $k \cdot c(x_F, x_I^t)$
  - Payoffs are realized



- Equilibrium concept: SPNE
- (PBE under private information considerations)

Model (5)

- Timing:



Analysis

Preliminaries

- Overview of the analysis:

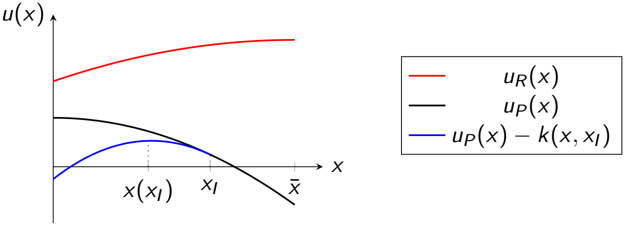
1. Optimal realized scale if resolution stage starts
2. Responder willingness to accept an offer
3. Proposer's decision on whether to start the resolution stage
4. Equilibrium

Realized scale at the Resolution Stage

- Realized scale at resolution stage is lower than intended scale

- Intended scale: x_I
- Realized scale: $x(x_I, k) \leq x_I$

$$x(x_I, k) \equiv \arg \max_{x \in [0,\bar{x}]} u_P(x) - kc(x, x_I) \tag{1}$$



Responder's willingness to accept the offer

- R's expected loss of rejecting an offer (if P does not start RS):

$$V_R(x_I^t, k) = p \cdot u_R(x(x_I, k)) + (1 - p) \cdot \delta V_R(x_I^{t+1}, k)$$

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- Suppose (for now) stationary x_I

  R's continuation loss of rejecting an offer at t:

$$V_R(x_I, k) = \begin{cases} \tilde{p} u_R(x(x_I, k)) & \text{if P does not start RS,} \\ u_R(x(x_I, k)) & \text{if P starts RS.} \end{cases}$$

- Composed probability of the resolution stage:

$$\tilde{p} = \frac{p}{1 - (1-p)\delta}$$

Responder's willingness to accept the offer

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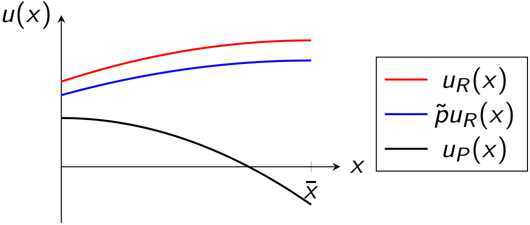
- R accepts the offer $y(k, \tilde{p})$ if: $y \leq V_R(x_I, k)$



Proposer's decision on whether to start the RS

- P starts RS if:

1. The payoff at RS is higher than delaying the game (if x_I is stationary)

$$u_P(x) - kc(x, x_I) \geq p \cdot \Big[u_P(x) - kc(x, x_I)\Big] + (1-p) \cdot \delta V_R(x(x_I, k)),$$

2. x_I is stationary:

$$u_P(x) - kc(x, x_I) \geq p \cdot \Big[u_P(x) - kc(x, x_I)\Big] + (1-p) \cdot \delta V_P(x_I^{t+1}, k),$$

    - Scaling-down x_I and continue negotiating is never optimal
    - Increasing x_I might be beneficial

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   - Scaling-down x_I and continue negotiating is never optimal
   - Increasing x_I might be beneficial

Equilibrium

- Result (efficiency): In equilibrium, the proposer makes an offer that the responder accepts at $t = 1$
- Equilibrium (candidates):
  - Deterministic threat: P starts the resolution stage,
  - Probabilistic threat: P waits for the shock (stationary)

Equilibrium

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- Equilibrium (candidates):
  - Deterministic threat: P starts the resolution stage,
  - Probabilistic threat: P waits for the shock (stationary)
- Equilibrium depends on two elements:
  1. Composed probability of the resolution stage:

$$\tilde{p} = \frac{p}{1 - (1 - p)\delta}$$

  2. Scaling-down cost: $k \cdot c(x_I^t, x_I^{t-1})$

Proposer's decision on whether to start the RS

- Define:

$$\tilde{p}_H = \frac{u_P(0)}{\delta u_R(0)} \quad \text{and} \quad \tilde{p}_L = \frac{u_P(0)}{\delta u_R(\bar{x})} \ .$$

Proposition

There exists a k^* such that, the equilibrium features a deterministic threat if:

- $\tilde{p} < \tilde{p}_L$, *or*
- $\tilde{p} \in [\tilde{p}_L, \tilde{p}_H]$ *and* $k \leq k^*$.

And a probabilistic threat if:

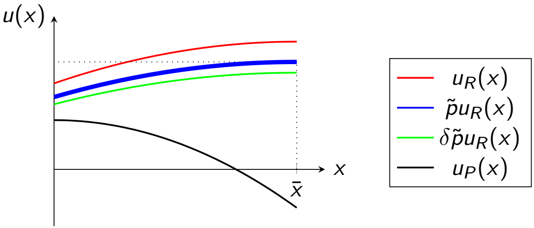
- $\tilde{p} > \tilde{p}_H$, *or*
- $\tilde{p} \in [\tilde{p}_L, \tilde{p}_H]$ *and* $k > k^*$.

1. High composed probability of exogenous shock

- If $\tilde{p} > \tilde{p}_H$: Probabilistic Threat
- P does not start resolution stage after a rejection

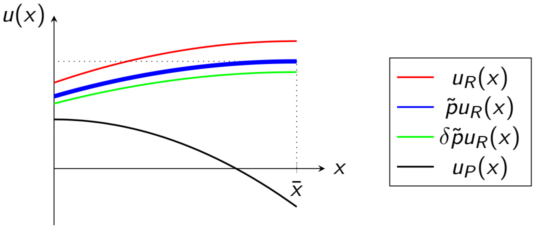
- Condition 1 is not satisfied:

  For any $x_I \in [0, \bar{x}]$: $u_P(x) - kc(x, x_I) < \delta \tilde{p} u_R(x)$



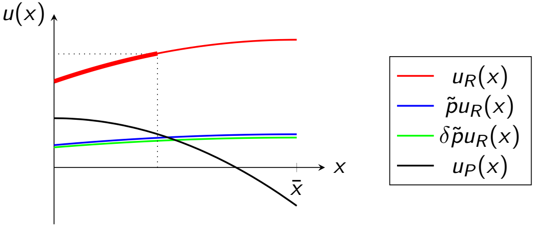
1. High composed probability of exogenous shock

- P relies on exogenous shock
- At $t = 1$ the proposer chooses $x_I = \bar{x}$ and offers: $\tilde{p}u_R(x(\bar{x}, k))$
- If scaling-down cost k is *high* $\Rightarrow$ Brinkmanship



2. Low composed probability of exogenous shock

- If $\tilde{p} < \tilde{p}_H$: Deterministic Threat
- P starts resolution stage after a rejection



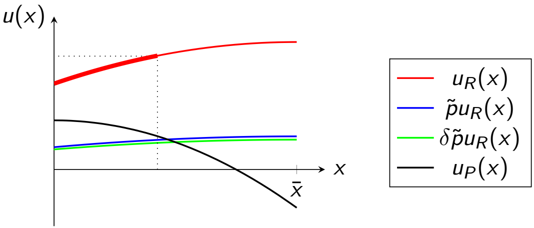
2. Low composed probability of exogenous shock

- For any k, there exists x_I^* such that:

$$(1) \quad u_P(x) - kc(x, x_I^*) \geq \delta \tilde{p} u_R(x(x_I^*, k))$$

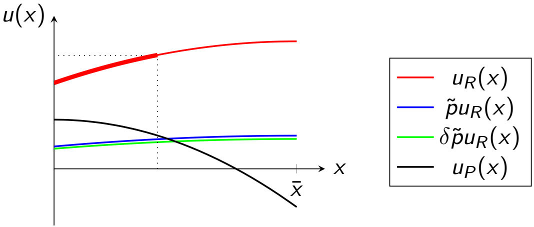
$$(2) \quad u_P(x) - kc(x, x_I^*) \geq \delta V_P(x_I^{t+1})$$

and $\quad u_R(x(x_I^*, k)) > \tilde{p} u_R(x(\bar{x}, k))$



2. Low composed probability of exogenous shock

- P chooses the highest possible x such that:
  starting the resolution stage is better than waiting



- In period $t = 1$ the proposer chooses:

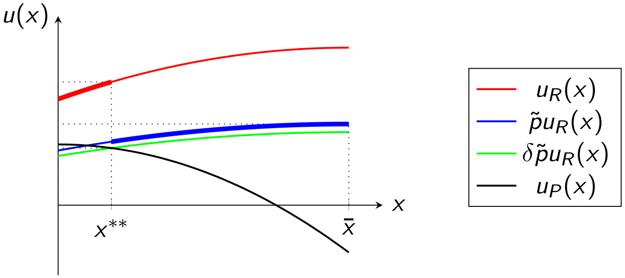
$$x_I(k) = \begin{cases} \bar{x} & \text{if } k \leq \underline{k} \\ x_I \text{ such that } u_P(x(x_I, k)) - kc(x(x_I, k), x_I) = \delta\tilde{p}u_R(x(\bar{x}, k)) & \text{if } k > \underline{k} \end{cases}$$

- And offers: $u_R(x(x_I, k))$

3. Intermediate composed probability of exogenous shock

- If $k \geq k^*$ $\Rightarrow$ Probabilistic threat
- If $k < k^*$ $\Rightarrow$ Deterministic threat

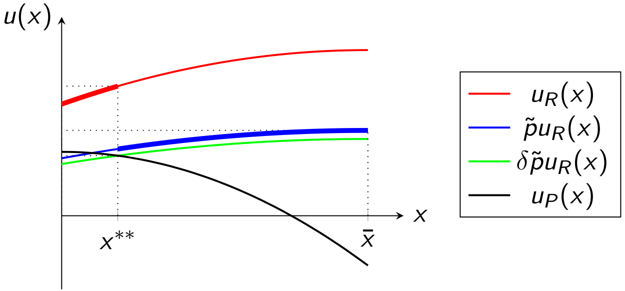
- Commitment Trap: It might be that P is better off with low scaling-down costs



3. Intermediate composed probability of exogenous shock

- Highest offer R is willing to accept: $u_R(x^{**})$
- For $x \in [0, x^{**}]$, higher offer under deterministic threat

- Commitment trap: $x \in [0, x^{**}]$ only feasible for low k
  - P wants to choose x_I, to induce deterministic threat.
  - It is not sequentially rational if k is high.



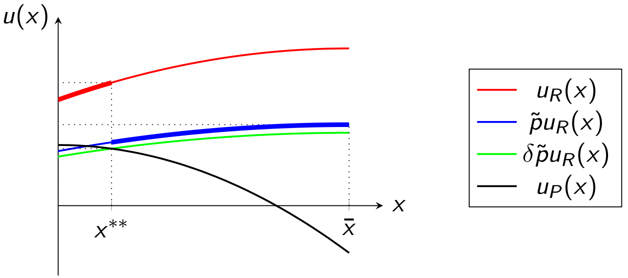
Commitment Trap

- If k is high ($k > k^*$), and realized scale is x^{**}, after a rejection:

$$u_P(x^{**}) - kc(x^{**}, x_I) < p \cdot \Big(u_P(x^{**}) - kc(x^{**}, x_I) \Big) + (1-p) \cdot \delta \tilde{p} u_R(x(\bar{x}, k)).$$

- Condition 2 (stationarity) is not satisfied:

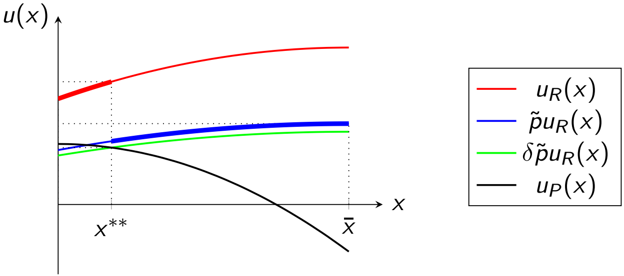
$$u_P(x^{**}) - kc(x^{**}, x_I) < \delta \tilde{p} u_R(x(\bar{x}, k)).$$



Commitment Trap

- But is it optimal? No

$$y = p \cdot u_R(x^{**}) + (1-p) \cdot \delta \tilde{p} u_R(x(\bar{x}, k)) < \tilde{p} u_R(x(\bar{x}, k))$$



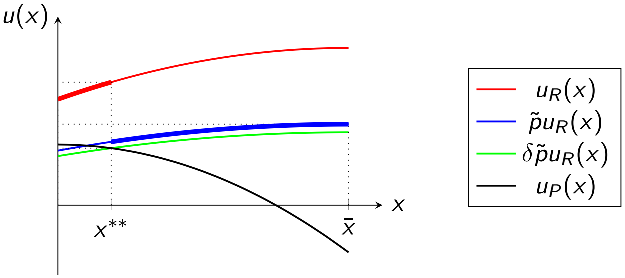
3. Intermediate composed probability of exogenous shock

- In period $t = 1$ the proposer chooses:

$$x_I(k) = \begin{cases} \bar{x} & \text{if } k \leq \underline{k} \\ x_I \text{ such that } u_P(x(x_I,k)) - kc(x(x_I,k),x_I) = \delta\tilde{p}u_R(x(\bar{x},k)) & \text{if } k \in [\underline{k}, k^*] \\ \bar{x} & \text{if } k > k^* \end{cases}$$

- And offers:

$$y = \begin{cases} u_R(x(x_I,k)) & \text{if } k \leq k^* \\ \tilde{p}u_R(x(x_I,k)) & \text{if } k > k^* \end{cases}$$



Commitment Trap

Proposition

If $\tilde{p} \in [\tilde{p}_L, \tilde{p}_H]$, the proposer induces the responder to accept a higher offer for any $k < k^*$ than for any $k \geq k^*$.

- Note: For $k \in [0, k^*]$ and $k > k^*$, a higher k induces the responder to accept a higher offer

Wrapping up

- Fixing δ:
- If p is high $\Rightarrow$ Probabilistic threat
  - If scaling-down are low $\Rightarrow$ Brinkmanship
- Example: Missiles crisis
- If p is low $\Rightarrow$ Deterministic threat
- Example: Picket lines
- If p is intermediate $\Rightarrow$ Commitment trap
- Example: Kyle Rittenhouse case

Private information

Rational behavioral type

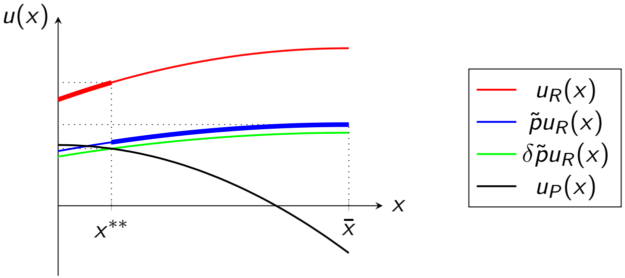
- Suppose it might be that the proposer never changes the scale
- Does the proposer can avoid the commitment trap?
  - Only if it is very probable that the proposer is behavioral type.
- Existing work: Abreu and Gul (2000), Sanktjohanser (2020)

Model

- Proposer type is proposer's private information
- Two rational proposer types (α):
  - Behavioral type ($\alpha = B$): does not change the scale
  - Normal type ($\alpha = N$): scaling-down costs given by k
- Responder's prior belief $P(\alpha = B) = \theta$
- Equilibrium selection: Normal type payoff maximizing equilibrium

Equilibrium

- Consider $\tilde{p} \in [\tilde{p}_L, \tilde{p}_H]$



Equilibrium

- Preliminaries:

- Idea: normal-type pretends to be the behavioral type

- Off the equilibrium path:
  - Behavioral type starts resolution stage
  - Normal type's decision depends on k

- Result (efficiency): In equilibrium, the proposer makes an offer that the responder accepts at $t = 1$

Equilibrium

- Result (informal): There exists $\bar{\theta}$ and $\underline{\theta}$ such that:

1. If $\theta > \bar{\theta}$, the equilibrium is pooling for any k:

$$x_I^N = x^{**} \quad \text{and} \quad x_I^B = x^{**}$$

- Responder is willing to accept

$$y = \theta u_R(x^{**}) + (1-\theta) V_R(x^{**})$$

- In this case: $y > V_R(x_I)$

- Commitment trap disappears

Equilibrium

2. If $\theta < \underline{\theta}$, the equilibrium is separating for any k:

$$x_I^N = x_I(k) \quad \text{and} \quad x_I^B \text{ is such that } u_R(x_I^B) = \tilde{p} u_R(x(x_I^N, k))$$

- Suppose pooling: R is willing to accept with

$$y = \theta u_R(x^{**}) + (1 - \theta) V_R(x^{**})$$

- In this case $y < V_R(x_I)$
- Commitment trap as in public info case

Equilibrium

3. If $\theta \in [\underline{\theta}, \bar{\theta}]$, there exist $k^{**}(\theta) > k^*$ such that:
   - Equilibrium is pooling if $k < k^{**}$
   - Equilibrium is separating if $k \geq k^{**}$

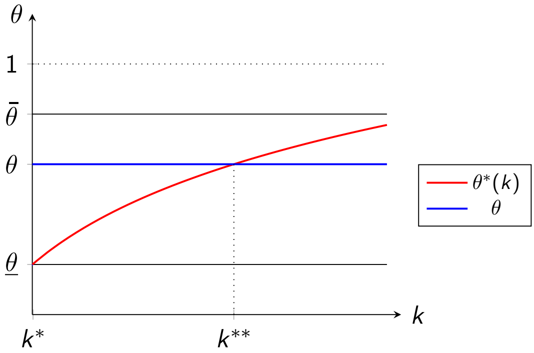
- Suppose pooling: R is willing to accept with

$$y = \theta u_R(x^{**}) + (1 - \theta) V_R(x^{**})$$

- If k is low: $y < u_R(x(x_I, k))$

- Commitment trap disappears for $k \in [k^*, k^{**}]$

Equilibrium



Concluding Remarks

Concluding Remarks

- Commitment to a threat in adversarial bargaining
- There is a risk of a welfare-destroying conflict

- Takeaway 1: Characterization of the threat
  - When to rely on the risk and when to commit to starting the conflict
  - Optimal threat scale

- Takeaway 2: Commitment trap
  - High pre-commitment power might be detrimental

- Takeaway 3: Does private information solve the commitment trap?
  - Only if the probability of being the behavioral type is very high

Thank you!