Threat, Commitment, and Brinkmanship in Adversarial Bargaining

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Motivation

- **Adversarial bargaining:**

  A party demands payment from another under threat of a welfare-destroying action

- Examples:
  - International conflict: demand of territory under threat of invasion
  - Plea bargaining: prosecutor threatens to take case to trial
  - Labor negotiation: better conditions or strike

Motivation

- **Adversarial bargaining:**

  A party demands payment from another under threat of a welfare-destroying action

- Examples:
  - International conflict: demand of territory under threat of invasion
  - Plea bargaining: prosecutor threatens to take case to trial
  - Labor negotiation: better conditions or strike

- **Theoretical questions:**
  - When is the threat credible?
  - Does commitment power to the scale of the action increase credibility?

- **Experimental/behavioral question:**
  - Do subjects use the commitment power?

Preview of the results / Contributions

- Theoretical contribution:
  - Characterization of credibility of the threat
  - Commitment trap: high commitment power can be detrimental
    - ⋆ Commitment power to the scale of the action
- Experimental contribution:
  - Subjects do take an irreversible action to *tie their hands*
  - Although it does not translate to higher offers accepted

Existing work

- Pre-commitment: mostly to a demand
  - Schelling (1956, 1960, 1966), Crawford (1982), Muthoo (1992, 1996), Dutta (2012, 2021)
- Reputational bargaining: behavioral type
  - Abreu and Gul (2000), Kambe (1999), Abreu and Sethi (2003), Wolitzky (2012), Atakan and Ekmekci (2014), Sanktjohanser (2020), Ekmekci and Zhang (2021)
- Applications
  - Schwarz and Sonin (2008)

Model

Model

- Two players:

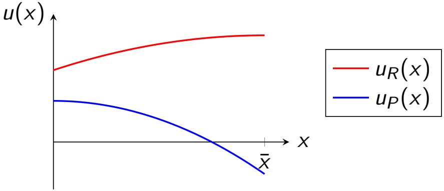
  Proposer (P) offers a deal to Responder (R)

- Multiperiod: $t \in \mathbb{N} \equiv \{1, 2, ...\}$
- Discount factor δ

- New elements:
  - A one-time conflict can start at the end of each period
  - Conflict ends the game
  - Surplus (pie) is negative and determined by the conflict

Model: Conflict

- Welfare-destroying conflict:
  - Payoffs are given by proposer's *scale* $x \in [0, \bar{x}]$:
  - P's payoff: $u_P(x)$
  - R's loss: $u_R(x)$



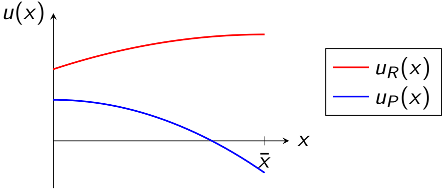
Model: Timing

- At each t:
  - P chooses a *intended scale* $x_I^t \in [0, \bar{x}]$
  - P offers a deal y to R
  - If accepted: $u_P = y$ and $u_R = -y$
  - If rejected: P decides whether to start the resolution stage
- If P decides no resolution stage: resolution stage starts with prob. p
- Scaling-down costs: $k \cdot c(x_I^t, x_I^{t-1})$
  - Increasing in the magnitude of the reduction: $(x_I^{t-1} - x_I^t)$
  - $c(x_I^t, x_I^{t-1}) = 0$ if $x_I^t \geq x_I^{t-1}$
  - k: Commitment power

Model: If conflict starts

- One-shot event
- Payoffs depend on *realized scale*

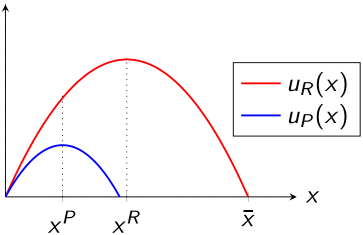
- P chooses *realized scale* x_F, at a cost $k \cdot c(x_F, x_I^t)$
- Payoffs are realized



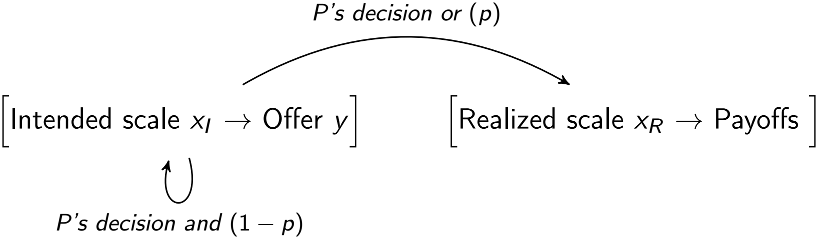
- Equilibrium concept: SPNE

Model: Assumption is WLOG

- Payoff specification of the conflict is without loss of generality
- Example:



Model: Wrapping up



Analysis

Preliminaries

- Overview of the analysis:

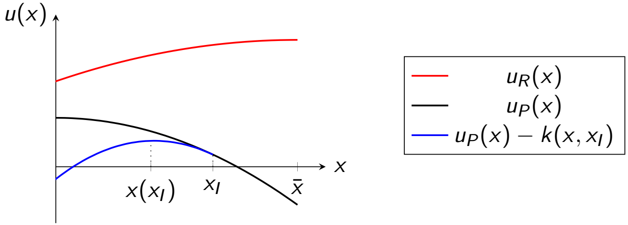
1. Optimal realized scale if resolution stage starts
2. Responder willingness to accept an offer
3. Proposer's decision on whether to start the resolution stage
4. Equilibrium

Realized scale at the Resolution Stage

- Realized scale at resolution stage is lower than intended scale

- Intended scale: x_I
- Realized scale: $x(x_I, k) \leq x_I$

$$x(x_I, k) \equiv \arg\max_{x \in [0, \bar{x}]} u_P(x) - kc(x, x_I) \qquad (1)$$



Responder's willingness to accept the offer

- R's expected loss of rejecting an offer (if P does not start RS):

$$V_R(x_I^t, k) = p \cdot u_R(x(x_I, k)) + (1-p) \cdot \delta V_R(x_I^{t+1}, k)$$

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- Suppose (for now) stationary x_I

  R's continuation loss of rejecting an offer at t:

$$V_R(x_I, k) = \begin{cases} \tilde{p} u_R(x(x_I, k)) & \text{if P does not start RS,} \\ u_R(x(x_I, k)) & \text{if P starts RS.} \end{cases}$$

- Composed probability of the resolution stage:

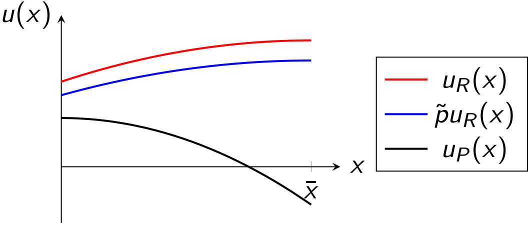
$$\tilde{p} = \frac{p}{1-(1-p)\delta}$$

Responder's willingness to accept the offer

- R's continuation loss of rejecting an offer at t:

$$V_R(x_I, k) = \begin{cases} \tilde{p} u_R(x(x_I, k)) & \text{if P does not starts RS,} \\ u_R(x(x_I, k)) & \text{if P starts RS.} \end{cases}$$

- R accepts the offer $y(k, \tilde{p})$ if: $y \leq V_R(x_I, k)$



Proposer's decision on whether to start the RS

- P starts RS if:

1. The payoff at RS is higher than delaying the game (if x_I is stationary)

$$u_P(x) - kc(x, x_I) \geq p \cdot \Big[u_P(x) - kc(x, x_I)\Big] + (1-p) \cdot \delta V_R(x(x_I, k)),$$

2. x_I is stationary:

$$u_P(x) - kc(x, x_I) \geq p \cdot \Big[u_P(x) - kc(x, x_I)\Big] + (1-p) \cdot \delta V_P(x_I^{t+1}, k),$$

   - Scaling-down x_I and continuing negotiating is never optimal
   - Increasing x_I might be beneficial

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   - Scaling-down x_I and continue negotiating is never optimal
   - Increasing x_I might be beneficial

Equilibrium

- Result (efficiency): In equilibrium, the proposer makes an offer that the responder accepts at $t = 1$
- Equilibrium (candidates):
  - Deterministic threat: P starts the resolution stage,
  - Probabilistic threat: P waits for the shock (stationary)

Equilibrium

- Result (efficiency): In equilibrium, the proposer makes an offer that the responder accepts at $t = 1$
- Equilibrium (candidates):
  - Deterministic threat: P starts the resolution stage,
  - Probabilistic threat: P waits for the shock (stationary)
- Equilibrium depends on two elements:
  1. Composed probability of the resolution stage:

  $$\tilde{p} = \frac{p}{1 - (1 - p)\delta}$$

  2. Scaling-down cost: $k \cdot c(x_I^t, x_I^{t-1})$

Proposer's decision on whether to start the RS

- Define:

$$\tilde{p}_H = \frac{u_P(0)}{\delta u_R(0)} \quad \text{and} \quad \tilde{p}_L = \frac{u_P(0)}{\delta u_R(\bar{x})} \ .$$

Proposition

There exists a k^* such that, the equilibrium features a deterministic threat if:

- $\tilde{p} < \tilde{p}_L$, *or*
- $\tilde{p} \in [\tilde{p}_L, \tilde{p}_H]$ *and* $k \leq k^*$.

And a probabilistic threat if:

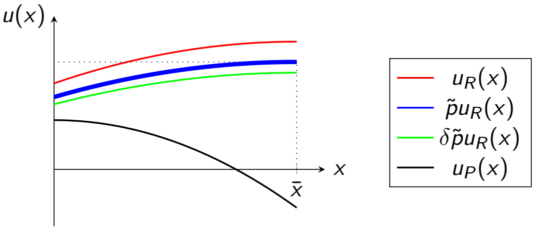
- $\tilde{p} > \tilde{p}_H$, *or*
- $\tilde{p} \in [\tilde{p}_L, \tilde{p}_H]$ *and* $k > k^*$.

1. High composed probability of exogenous shock

- If $\tilde{p} > \tilde{p}_H$: Probabilistic Threat
- P does not start resolution stage after a rejection

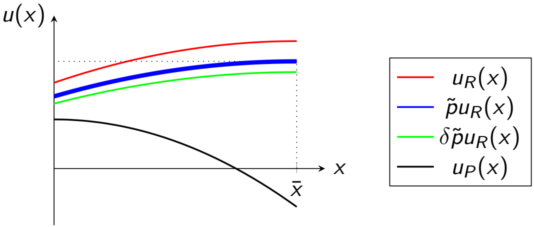
- Condition 1 is not satisfied:

  For any $x_I \in [0, \bar{x}]$: $u_P(x) - kc(x, x_I) < \delta \tilde{p} u_R(x)$



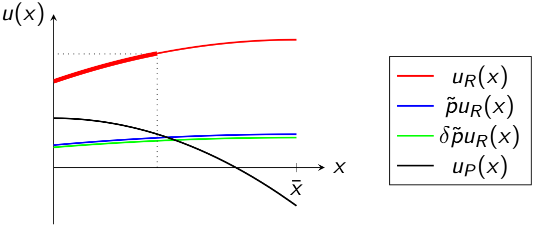
1. High composed probability of exogenous shock

- P relies on exogenous shock
- At $t = 1$ the proposer chooses $x_I = \bar{x}$ and offers: $\tilde{p}u_R(x(\bar{x}, k))$
- If scaling-down cost k is *high* $\Rightarrow$ Brinkmanship



2. Low composed probability of exogenous shock

- If $\tilde{p} < \tilde{p}_H$: Deterministic Threat
- P starts resolution stage after a rejection



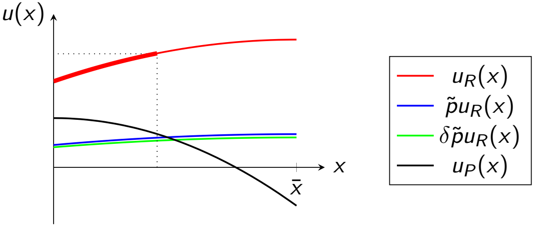
2. Low composed probability of exogenous shock

- For any k, there exists x_I^* such that:

$$(1) \quad u_P(x) - kc(x, x_I^*) \geq \delta \tilde{p} u_R(x(x_I^*, k))$$

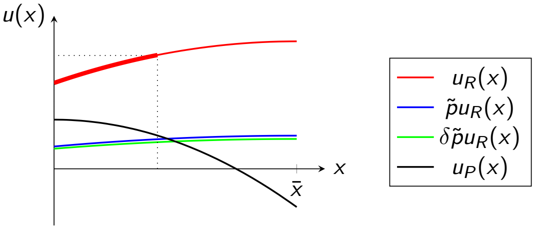
$$(2) \quad u_P(x) - kc(x, x_I^*) \geq \delta V_P(x_I^{t+1})$$

and $\quad u_R(x(x_I^*, k)) > \tilde{p} u_R(x(\bar{x}, k))$



2. Low composed probability of exogenous shock

- P chooses the highest possible x such that:
  starting the resolution stage is better than waiting



- In period $t = 1$ the proposer chooses:

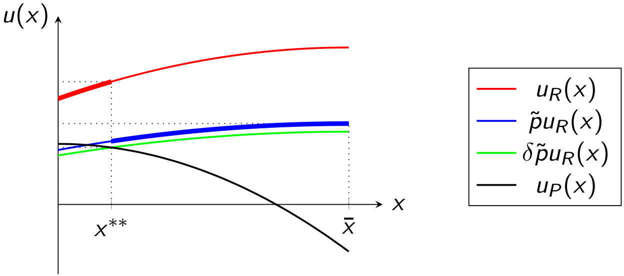
$$x_I(k) = \begin{cases} \bar{x} & \text{if } k \leq \underline{k} \\ x_I \text{ such that } u_P(x(x_I, k)) - kc(x(x_I, k), x_I) = \delta\tilde{p}u_R(x(\bar{x}, k)) & \text{if } k > \underline{k} \end{cases}$$

- And offers: $u_R(x(x_I, k))$

3. Intermediate composed probability of exogenous shock

- If $k \geq k^*$ $\Rightarrow$ Probabilistic threat
- If $k < k^*$ $\Rightarrow$ Deterministic threat

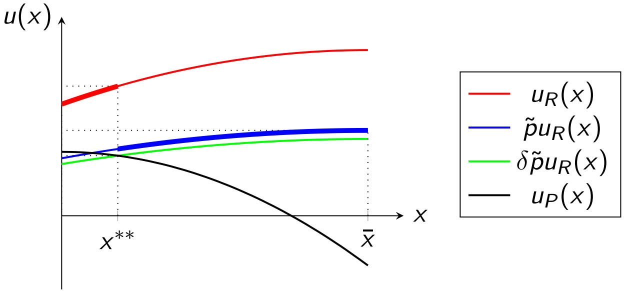
- Commitment Trap: It might be that P is better off with low scaling-down costs



3. Intermediate composed probability of exogenous shock

- Highest offer R is willing to accept: $u_R(x^{**})$
- For $x \in [0, x^{**}]$, higher offer under deterministic threat

- Commitment trap: $x \in [0, x^{**}]$ only feasible for low k
  - P wants to choose x_I, to induce deterministic threat.
  - It is not sequentially rational if k is high.



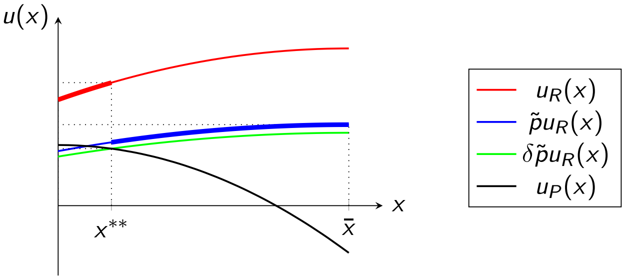
Commitment Trap

- If k is high ($k > k^*$), and realized scale is x^{**}, after a rejection:

$$u_P(x^{**}) - kc(x^{**}, x_I) < p \cdot \Big(u_P(x^{**}) - kc(x^{**}, x_I) \Big) + (1-p) \cdot \delta \tilde{p} u_R(x(\bar{x}, k)).$$

- Condition 2 (stationarity) is not satisfied:

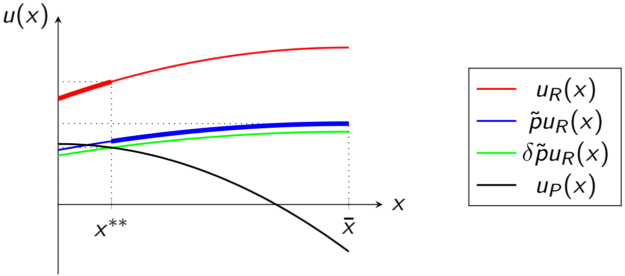
$$u_P(x^{**}) - kc(x^{**}, x_I) < \delta \tilde{p} u_R(x(\bar{x}, k)).$$



Commitment Trap

- But is it optimal? No

$$y = p \cdot u_R(x^{**}) + (1-p) \cdot \delta \tilde{p} u_R(x(\bar{x},k)) < \tilde{p} u_R(x(\bar{x},k))$$



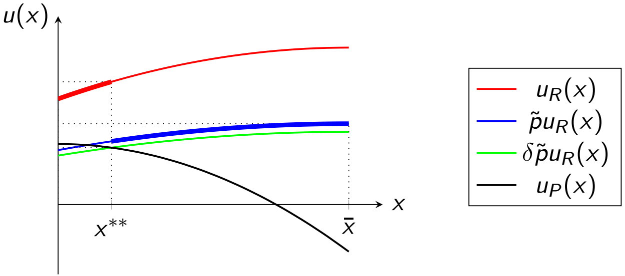
3. Intermediate composed probability of exogenous shock

- In period $t = 1$ the proposer chooses:

$$x_I(k) = \begin{cases} \bar{x} & \text{if } k \leq \underline{k} \\ x_I \text{ such that } u_P(x(x_I, k)) - kc(x(x_I, k), x_I) = \delta \tilde{p} u_R(x(\bar{x}, k)) & \text{if } k \in [\underline{k}, k^*] \\ \bar{x} & \text{if } k > k^* \end{cases}$$

- And offers:

$$y = \begin{cases} u_R(x(x_I, k)) & \text{if } k \leq k^* \\ \tilde{p} u_R(x(x_I, k)) & \text{if } k > k^* \end{cases}$$



Commitment Trap

Proposition

If $\tilde{p} \in [\tilde{p}_L, \tilde{p}_H]$, the proposer induces the responder to accept a higher offer for any $k < k^*$ than for any $k \geq k^*$.

- Note: For $k \in [0, k^*]$ and $k > k^*$, a higher k induces the responder to accept a higher offer

Wrapping up

- Fixing δ:
- If p is high $\Rightarrow$ Probabilistic threat
  - If scaling-down are low $\Rightarrow$ Brinkmanship
- Example: Missiles crisis
- If p is low $\Rightarrow$ Deterministic threat
- Example: Picket lines
- If p is intermediate $\Rightarrow$ Commitment trap
- Example: Kyle Rittenhouse case

Experimental Question:

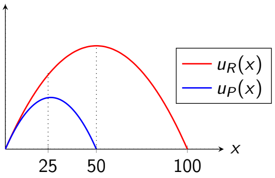
Do subjects take advantage of the commitment power?

Simplified Model

- One bargaining round, and then conflict
- Timing
  - Proposer chooses x^I
  - Proposer offers a deal
  - If accepted: game ends
  - If rejected, proposer chooses $x^F \in [\alpha x^I, x^I]$, $\alpha \in (0, 1)$
  - Payoff are realized according to *conflict's payoff*

Conflict and commitment (experimental design)

- $x \in [0, 100]$
- Responder's loss: $\mathbb{E}u_R = x \cdot \frac{100-x}{100}$
- Proposer's payoff: $\mathbb{E}u_P = x \cdot \frac{100-x}{100} - x \cdot \left(1 - \frac{100-x}{100}\right)$
- Commitment: $\alpha_H = 0.7$ and $\alpha_L = 0.2$
- Plot:



Results:

- Predictions

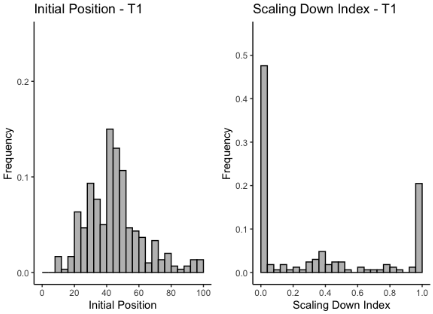
	$\alpha = 0.7$	$\alpha = 0.2$
Initial threat	71.4	[25,100]
Offer	35	28.7
Final threat	50	25

- Results:

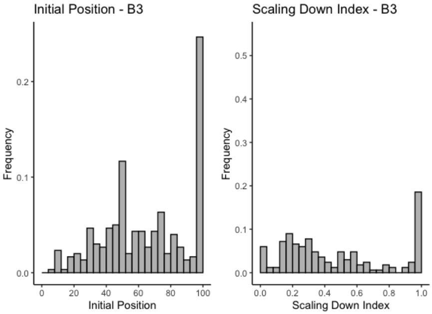
	$\alpha = 0.7$	$\alpha = 0.2$
Mean initial threat	44.8	65.1
Mean offer	35.8	38.1
Proportion of accepted offers	0.45	0.44
Mean final threat	38.3	33.9

Results:

- Subjects do use the commitment power to *tie-their-hands*



a $\alpha = 0.7$



b $\alpha = 0.2$

Concluding Remarks

Concluding Remarks

- Role of commitment in adversarial bargaining
- Takeaway 1: Characterization of the threat
  - When to rely on the risk and when to commit to starting the conflict
  - Optimal threat scale
- Takeaway 2: Commitment trap in multiperiod bargaining
  - Novel theoretical result
  - High commitment power might be detrimental
- Takeaway 3: Do subjects actually use the commitment power?
  - Experimentally tested
  - Yes, but it does not translate to gaining bargaining power

Thank you!

Private information

Rational behavioral type

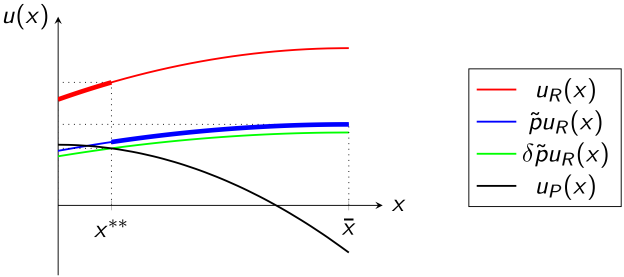
- Suppose it might be that the proposer never changes the scale
- Does the proposer can avoid the commitment trap?
  - Only if it is very probable that the proposer is behavioral type.
- Existing work: Abreu and Gul (2000), Sanktjohanser (2020)

Model

- Proposer type is proposer's private information
- Two rational proposer types (α):
  - Behavioral type ($\alpha = B$): does not change the scale
  - Normal type ($\alpha = N$): scaling-down costs given by k
- Responder's prior belief $P(\alpha = B) = \theta$
- Equilibrium selection: Normal type payoff maximizing equilibrium

Equilibrium

- Consider $\tilde{p} \in [\tilde{p}_L, \tilde{p}_H]$



Equilibrium

- Preliminaries:

- Idea: normal-type pretends to be the behavioral type

- Off the equilibrium path:
  - Behavioral type starts resolution stage
  - Normal type's decision depends on k

- Result (efficiency): In equilibrium, the proposer makes an offer that the responder accepts at $t = 1$

Equilibrium

- Result (informal): There exists $\bar{\theta}$ and $\underline{\theta}$ such that:

1. If $\theta > \bar{\theta}$, the equilibrium is pooling for any k:

$$x_I^N = x^{**} \quad \text{and} \quad x_I^B = x^{**}$$

- Responder is willing to accept

$$y = \theta u_R(x^{**}) + (1-\theta) V_R(x^{**})$$

- In this case: $y > V_R(x_I)$

- Commitment trap disappears

Equilibrium

2. If $\theta < \underline{\theta}$, the equilibrium is separating for any k:

$$x_I^N = x_I(k) \quad \text{and} \quad x_I^B \text{ is such that } u_R(x_I^B) = \tilde{p} u_R(x(x_I^N, k))$$

- Suppose pooling: R is willing to accept with

$$y = \theta u_R(x^{**}) + (1 - \theta) V_R(x^{**})$$

- In this case $y < V_R(x_I)$
- Commitment trap as in public info case

Equilibrium

3. If $\theta \in [\underline{\theta}, \bar{\theta}]$, there exist $k^{**}(\theta) > k^*$ such that:
    - Equilibrium is pooling if $k < k^{**}$
    - Equilibrium is separating if $k \geq k^{**}$

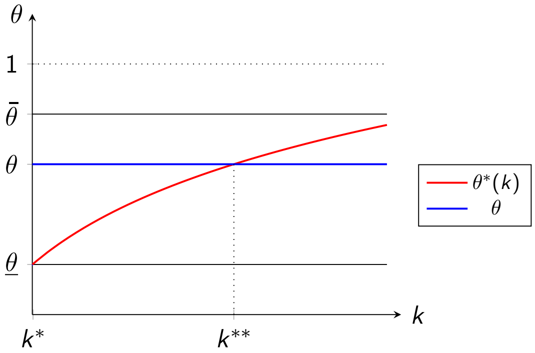
- Suppose pooling: R is willing to accept with

$$y = \theta u_R(x^{**}) + (1 - \theta) V_R(x^{**})$$

- If k is low: $y < u_R(x(x_I, k))$

- Commitment trap disappears for $k \in [k^*, k^{**}]$

Equilibrium



Concluding Remarks

- Takeaway 4: Does private information solve the commitment trap?
  - Only if the probability of being the behavioral type is very high

Thank you!