Voluntary Disclosure of Evidence in Plea Bargaining

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June 2022

Abstract

I study how voluntary disclosure of information affects outcomes in plea bargaining. A prosecutor negotiates a sentence with a defendant who privately knows whether he is guilty or innocent. The prosecutor can gather evidence regarding the defendant's type during negotiations, and a trial assigns payoff depending on the evidence if they fail to reach an agreement. Voluntary disclosure induces endogenous second-order belief uncertainty. I show that a purely sentence-motivated prosecutor might disclose exculpatory evidence and that voluntary disclosure generates inefficient outcomes. Mandatory disclosure is socially preferable as outcomes are fairer and efficient. The prosecutor is better off under mandatory disclosure.

Keywords: Bargaining, plea bargain, second-order belief uncertainty, disclosure.
JEL classifications: C78, D82, D83, K40

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1 Introduction

1.1 Motivation and Results

In many bargaining situations, one party can gather information and keep the outcome private. A real estate agent who inspects a house for sale might decide to conceal information that would increase the price of the house. After reviewing a company, an investor might only reveal information that increases the return rate that she is asking. In plea bargaining (the context I examine in this paper), a prosecutor who finds exculpatory evidence might hide it from the defendant. I study the disclosure decision of the newly informed party when the disclosure of new information is voluntary. I also examine whether mandatory, instead of voluntary, disclosure of information is socially desirable.

Plea bargaining is a case of particular relevance in the US criminal system, in which more than 90% of criminal cases end in plea bargaining instead of a trial.[1] In plea bargaining, a prosecutor and a defendant negotiate for a sentence to avoid trial. During the negotiation, the prosecutor can search for evidence regarding the culpability of the defendant. In many circuit courts in the US, the disclosure of evidence is voluntary during plea bargaining; hence, the prosecutor can hide exculpatory evidence during the negotiation but must disclose it at trial. If the prosecutor wants the judge to assign as high a sentence as possible, will she disclose exculpatory evidence? Even if she discloses it, is it socially desirable to impose mandatory disclosure of evidence in plea bargaining?

To answer these questions, I study a dynamic plea-bargaining model between a prosecutor (she) and a defendant (he). The defendant's type can be innocent or guilty, and the defendant is privately informed about his type. The prosecutor has inconclusive default evidence at the beginning of the game and can decide whether to investigate for new conclusive hard evidence that can be exculpatory if the defendant is innocent or incriminating if the defendant is guilty. The investigation process is not perfect; with some probability, the prosecutor will not find new evidence.

If the prosecutor finds new evidence, she can voluntarily disclose it to the defendant. After the disclosure decision, the prosecutor offers a sentence to the defendant. If the offer is accepted, the game ends; if not, a new period starts. If they do not reach an agreement during a finite number of periods, they go to trial. I model the trial as a rule that assigns a sentence depending on the evidence that the prosecutor gathered: Exculpatory evidence sets the defendant free, default evidence leads to a low sentence, and incriminating evidence leads to a high sentence.

The first main result shows that, in some cases, the prosecutor discloses exculpatory evidence. Suppose the prosecutor's prior belief about the defendant being guilty is low; if the prosecutor finds exculpatory evidence that exonerates the defendant, she hides it

[1] See Devers (2011).

and makes a low offer that the innocent defendant accepts. However, if the prior belief is high, she discloses exculpatory evidence and sets the defendant free in equilibrium.

The prosecutor is able to hide exculpatory evidence because she induces second-order belief uncertainty on the innocent defendant when she investigates for new evidence and the disclosure is voluntary. That is, the defendant does not know what evidence the prosecutor has. It implies that the innocent defendant is willing to accept a positive sentence because of the possibility of the prosecutor showing default evidence at the trial. To not reveal the evidence, when the prosecutor has exculpatory evidence, she needs to make the same offer she would have made if she had default evidence. It is optimal for the prosecutor to make that offer when the prior belief is low because the defendant will accept it. However, when the prior belief is high enough, the prosecutor's offer, if she had default evidence, is higher than what the innocent defendant is willing to accept. So, the prosecutor prefers to disclose the evidence because otherwise, the defendant will reject the offer, and they will go to trial, which is costly.

The second main result shows that mandatory disclosure of evidence is socially preferable to voluntary disclosure for the following reasons: First, from a utilitarian point of view, mandatory disclosure of evidence is efficient because an agreement is always reached during the plea bargaining process and the prosecutor never incurs the cost of going to trial. There is a positive probability of going to trial when the prior belief about the defendant being guilty is high enough with voluntary disclosure. Second, the prosecutor is better off with mandatory disclosure of evidence because hiding exculpatory evidence has a downside; she cannot extract the full surplus when she has default evidence. It produces a commitment effect; if the prosecutor can ex ante commit to disclose any evidence she found, she would do it. But with voluntary disclosure, this is not possible because if she gets exculpatory evidence, she will hide it if she can. Third, in the frequent offers limit—when the length of each period goes to zero—the innocent defendant is better off, and the guilty defendant is worse off with mandatory disclosure of evidence.

For both the voluntary and mandatory disclosure of evidence cases, the prosecutor and defendant reach an agreement during the first period when the prosecutor's prior belief about the defendant being guilty is low enough. It is because the prosecutor prefers not to investigate, and instead, to reach an agreement immediately. For higher values of the prior belief, there is a deadline effect. If the disclosure of evidence is voluntary, the deadline effect is as in Spier (1992): The prosecutor and defendant reach an agreement just at the deadline with a high probability compared to other periods, and in some cases, they do not reach an agreement and go to trial. If the disclosure of evidence is mandatory, they also have a higher probability of reaching an agreement at the deadline, but they never go to trial.

Outline: The plan of the paper is as follows. Section 1.2 provides background on the plea bargaining process, and Section 1.3 discusses the related literature. Section 2 introduces the model. Section 3 shows a one-period benchmark. Section 4 shows the general result for N periods. Section 5 discusses the frequent offer limit case. Section 6 describe the mandatory disclosure case, and Section 7 compares mandatory and voluntary disclosure case. Section 8 concludes, Appendix A provides extensions of the model, and Appendix B contains the proofs.

1.2 Plea Bargaining Background

In US criminal law, plea bargaining is the pretrial process in which the prosecutor and the defendant negotiate an agreement in which the defendant pleads guilty in exchange for a lower sentence.[2] This agreement, called a plea bargain, allows the prosecutor and the defendant to avoid a trial and the associated cost and uncertainty. If they do not reach an agreement, the case goes to trial. The role of the prosecutor is to represent society in the criminal case brought against the defendant.

During the trial, the prosecutor must disclose all of the evidence. It is because the trial is protected by the Brady Rule, named for *Brady v. Maryland (1963)*, which requires prosecutors to disclose materially exculpatory evidence in their possession to the defendant.[3] The Brady Rule is not always extended to the plea bargaining process. According to Casey (2020), the Brady Rule is applied to the plea bargaining process in the Seventh, Ninth, and Tenth Circuit courts, while it is not applied to pretrial negotiations in the First, Second, Fourth, and Fifth Circuits. State courts are divided in a similar fashion.[4]

Applying the Brady Rule to the plea bargaining process is a policy question that has attracted scholars and the media attention.[5] Some arguments in favor of extending the Brady Rule to plea bargaining are related to the knowing and voluntary nature of a guilty plea; failure to disclose materially exculpatory evidence precludes a knowing and voluntary guilty plea. As a consequence, the Brady Rule will likely reduce convictions of innocent defendants. Arguments against hold that extending the Brady Rule will result in higher costs and less efficiency.

The first main result of the paper shows that even when the Brady Rule does not apply to plea bargaining, the prosecutor might drop cases under certain circumstances. The second main result of the paper addresses whether the Brady Rule should apply during pretrial negotiations. I show that applying the Brady Rule to pretrial negotiations is desirable because the prosecutor and the defendant avoid costly trials by reaching an agreement. Also, the expected sentence is lower for the innocent defendant and higher

[2]In the US system, the judge has to agree with the plea bargain. In this paper, I assume the judge always agrees with it when the prosecutor and defendant agree.

[3]See *Brady v. Maryland*, 373 U.S. 83, 83 S. Ct. 1194, 10 L. Ed. 2d 215 (1963).

[4]There is no clear definition in the other Circuits courts.

[5]See Casey (2020); Daughety and Reinganum (2020); or Sanders (2019) for some references. See also a *New York Times* editorial, "Beyond the Brady Rule" https://www.nytimes.com/2013/05/19/opinion/sunday/beyond-the-brady-rule.html

for the guilty defendant, while the expected payoff for the prosecutor is higher with mandatory disclosure.

1.3 Related Literature

In my model, the prosecutor investigates seeking new evidence, and the voluntary disclosure generates second-order uncertainty on the defendant. Hence, this paper mainly relates to the literature on pretrial negotiations, bargaining with information arrival, and higher-order uncertainty in bargaining.

Pretrial negotiations: Spier (1992) and Fuchs and Skrzypacz (2013) present pretrial bargaining models with incomplete information and a deadline that includes a rule to assign payoffs. They show that many agreements occur just at the deadline. Although there is a similar deadline effect in my model, I also focus on the disclosure of information. Garoupa and Rizzolli (2011) study a model in which the prosecutor might decide not to investigate before trial and conclude that innocent defendants may be worse off with the Brady Rule at trial. Daughety and Reinganum (2018) present a trial model in which a prosecutor with career concerns can violate the Brady Rule at trial. These papers focus on modeling the trials, while I focus on the pretrial negotiation and model the trial in reduced form.

In the literature on plea bargaining, Landes (1971) examines how the probability of winning at trial affects pretrial negotiations. Grossman and Katz (1983) and Reinganum (1988) study the welfare effects of plea bargaining, depending on the probability of conviction at trial. Baker and Mezzetti (2001) examine a model in which the prosecutor can choose the costly precision of a signal about defendant type. Bjerk (2007) presents a model in which new information can be revealed at trial. Vasserman and Yildiz (2019) present a model in which negotiating parties are optimistic about the decision at trial and anticipate a possible arrival of public information before the trial date. None of these papers allow for the possibility of disclosing information or effects of the Brady Rule during pretrial negotiations. Ispano and Vida (2021) present a model of interrogations in which a law enforcer official and a suspect interchange messages regarding if the suspect is innocent or guilty, and the law enforcer may disclose previously acquired evidence. Their paper focuses on the optimal interrogation policy to learn the suspect type, while I focus on the bargaining over the sentence.

Bargaining with information arrival: Duraj (2020) considers a bargaining model in which the buyer can choose how accurately she learns about her valuation of a good being traded, and she can disclose the updated valuation. Esö and Wallace (2019) consider a bargaining model in which the value of the good that is being traded is exogenously and privately revealed and can be disclosed. They show that the possibility of learning might result in a delay in reaching an agreement. Esö and Wallace (2014) analyze the effect of exogenously having verifiable and unverifiable evidence in a one-period bargaining model and show that the proposer is always better off with verifiable evidence. Hwang and Li (2017) present a model in which the buyer's outside option stochastically arrives and can be disclosed by the seller. If the outside option is private information, the buyer prefers never to reveal it, and there is delay in the game. These papers show that each party with new information hides detrimental evidence and discloses beneficial evidence. In my model, the party with new information will disclose not just the beneficial information, but also the detrimental information to the other party. I characterize conditions under which doing so is optimal.

Daley and Green (2020); Fuchs and Skrzypacz (2010); Hwang (2018); Lomys (2017); Ortner (2017); and Ortner (2020) consider variations of the Coase conjecture model with arrival of new information (private or public). They do not consider disclosure of private information. The focus of the present paper is the possibility of disclosing information and how that affects the efficiency of the bargaining.

Higher-order uncertainty in bargaining: Feinberg and Skrzypacz (2005) study a bargaining model in which one party privately knows his type and the other party has a private belief about the type. This second-order uncertainty is exogenous, and there is no disclosure of information during the bargaining. The authors show that there is a delay in the agreement. In my model, the uncertainty is endogenous rather than exogenous, and one party can eliminate the uncertainty of the other party by revealing information.

Friedenberg (2019) studies an alternating-offer bargaining model in which delay in agreement may arise when players face strategic uncertainty—that is, uncertainty about the opponent's play. There is no strategic uncertainty in my model; instead, there is uncertainty in the second-order belief. Also, I focus on the disclosure decision.

Disclosure of verifiable information: Dye (1985) studies a model with a similar evidence structure to the present paper. The receiver is uncertain about the sender's information endowment, and if there is information and the sender discloses it, it perfectly reveals the state of the world. There are several extensions of Dye's (1985) model. The closest is Acharya et al. (2011), which includes a dynamic setting. Dye (1985) shows there is no disclosure of detrimental information, while Acharya et al. (2011) shows that disclosure of negative information only happens after negative public news is exogenously revealed. In my paper, the prosecutor voluntarily reveals detrimental information. Additionally, in the mentioned papers, the receiver is originally uninformed about the state of the world. In contrast, in my model, the defendant knows his type and the prosecutor investigates it.

2 Model

There are two players: a prosecutor (she) and a defendant (he). The prosecutor's only objective is to assign the highest possible sentence to the defendant, regardless of the defendant's innocence, while the defendant wants the lowest possible sentence.[6] The defendant is privately informed of his type α, which can be innocent ($\alpha = I$) or guilty ($\alpha = G$). The defendant's type is unknown to the prosecutor. Let $\theta \in (0, 1)$ denote the prior probability that the prosecutor assigns to $\alpha = G$. The game is divided into two phases: *plea bargaining* and *trial*. The game starts with the plea bargaining phase, in which the prosecutor can investigate for new evidence and try to reach an agreement with the defendant to avoid trial. They move to the trial phase only if they fail to reach an agreement before a deadline. The trial is a reduced-form function that assigns reward to the prosecutor and punishment to the defendant, depending on the prosecutor's evidence at that moment.

The plea bargaining phase ends at time $T > 0$. This phase is divided into $N \geq 1$ periods, with the length of each period equal to $\Delta = T/N$. The set of evidence that exists in this environment is $y \in \{e, d, h\}$, where $y = e$ stands for *exculpatory evidence*, $y = h$ for *incriminating evidence*, and $y = d$ for *default evidence*.

The prosecutor can investigate for new evidence at the beginning of each period, and she can voluntarily disclose the new evidence. At the end of each period, she makes an offer to the defendant. There is no discount factor or cost of delay during the plea bargaining phase for any player.

The timing within each period n is:

1. Investigation for new evidence: At the beginning of the game, the prosecutor has evidence $y = d$ and a prior belief $Pr(\alpha = G) = \theta \in (0, 1)$. At the start of each period $n = \{1, 2, ..., N\}$, the prosecutor can investigate to obtain more evidence. The probability of getting evidence follows an exponential distribution that depends on the length of each period; the probability of finding new evidence at each period n is equal to $1 - q^{\frac{1}{N}}$, where $q = e^{-\lambda}$ for $\lambda > 0$.[7]

The new evidence depends on the defendant's type: If $\alpha = I$ the investigation's

[6] This implies that even if the prosecutor knows the defendant is innocent, she still wants him to have the highest possible sentence. This should be interpreted as an extreme case, to show that (in the following Sections) even a purely sentence-motivated prosecutor will reveal exculpatory evidence. Although this is a simplification, many prosecutors seem to be motivated by high sentences rather than justice. Medwed (2004) notes that many prosecutors resist exonerating the innocent even when prisoners have presented overwhelming proof of their innocence. Also, Keenan et al. (2011) and Garrett (2017) argue that prosecutorial misconduct is a widespread problem in the U.S. and list cases in which prosecutors suppress exculpatory evidence at the trial. Finally, Pfaff (2017) argues that the criminal justice system provides incentives for prosecutors to seek an overly aggressive punishment, and Alschuler (2015) argues that the plea bargaining process tends to convict more innocent people than trials do.

[7] Note that $q^{\frac{1}{N}} = e^{-\lambda \frac{\Delta}{T}}$.

outcome belongs to $y^I \in \{\emptyset, e\}$; if $\alpha = G$ the outcome belongs to $y^G \in \{\emptyset, h\}$. The implication is that after getting $y = e$ or $y = h$, the prosecutor updates her belief to $\theta' = 0$ and $\theta' = 1$, respectively. The prosecutor gets new evidence only once, and it replaces default evidence. Note that the probability of finding new evidence is independent of the defendant's type; this implies that if the prosecutor does not get new evidence after the investigation, she does not update her belief about the defendant's type.

The decision to investigate and the outcome of the investigation are private information for the prosecutor. I say that the prosecutor is y-type if she has evidence $y \in \{e, d, h\}$.

2. Disclosure of new evidence: After the outcome is realized, the prosecutor can choose to disclose the new evidence to the defendant. I assume that only new evidence can be disclosed.[8] I also assume that the disclosure of evidence is voluntary during the plea bargaining phase, but it is mandatory at trial. I discuss the case with mandatory disclosure of evidence during the plea bargaining phase in Section 1.5.

3. Offer: After the prosecutor decides whether to disclose, the prosecutor makes an offer $x \in \mathbb{R}$ to the defendant. An offer is a sentence that assigns utility $u_D = -x$ to the defendant and utility $u_P = x$ to the prosecutor if it is accepted.[9] If the offer is accepted, the game ends, and if the offer is rejected and $n < N$, a new period $n+1$ starts. If the offer is rejected at $n = N$, they go to trial. Figure 1 shows the timing of a period n during the plea bargaining phase.

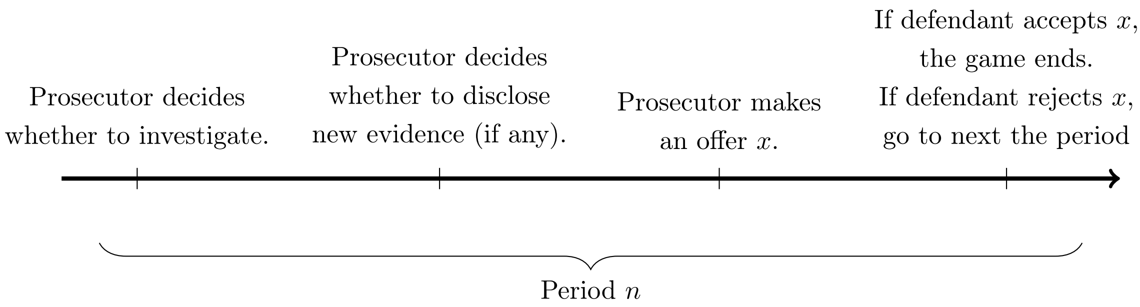

Figure 1: Timeline of a period n

This paper's focus is the plea bargaining phase. For this reason, I model the trial as a simple rule that assigns a sentence s to evidence.[10]

$$s = \begin{cases} 0 & \text{if } y = e \\ d & \text{if } y = d \\ h & \text{if } y = h. \end{cases}$$

[8] I assume the evidence is hard; the prosecutor cannot show what she does not have.

[9] The zero-sum nature of the payoff is without loss of generality.

[10] An alternative way to model the trial is assuming that the sentence depends on the public posterior belief θ^T. The trial assigns $\theta^T h$ instead of d if the evidence is neither e nor h. This alternative modeling does not change the key insights regarding disclosure as long as there is an investigation. However, the decision of whether to investigate is qualitatively different. I discussed this case in Appendix A.4.

where $0 < d < h$. The trial has a cost of c for the prosecutor.[11] Hence, the payoffs for the prosecutor and the defendant at the trial are: $u_D = s - c$ and $u_D = -s$.

I denote the expected payoff for the prosecutor as v^P, and I define *expected punishment* to be the absolute value of the expected payoff for the defendant. I denote the expected punishment as v^α, with $\alpha \in \{I, G\}$.

Parametric assumption: For simplicity, in all the results of the paper and its proofs, I restrict attention to the case when $c > d(1-q)$ and $h > d\big(q + \frac{c}{c-d(1-q)}\big)$. That is, the cost of the trial is not very small, and the payoff of getting incriminating evidence is large enough to be attractive. It provides clearer and simpler results and does not affect the message of the paper. I discuss the case these assumptions are not satisfied in Appendix A.5.

Histories and strategies. Call $\tilde{y} \in \{\emptyset, e, h\}$ the evidence disclosed by the prosecutor, where $\tilde{y} = \emptyset$ means that the prosecutor has not disclosed any evidence. At any period n before the agreement is reached, the prosecutor's history $h_n^P = \{y_n, \{\tilde{y}_s, x_s\}_{s \leq n}\}$ contains the evidence the prosecutor has, the disclosed evidence, and the offers she has made. The defendant's history $h_n^D = \{\alpha, \{\tilde{y}_s, x_s\}_{s \leq n}\}$ contains his type, the disclosed evidence, and the previous offers. A (pure) strategy for the prosecutor $\sigma^P : h_n^P \mapsto (\{\textit{investigation, no investigation}\}, \tilde{y}_n(y), x_n)$ maps prosecutor's history h_n^P to the decision to investigate or not, a disclosure decision after the outcome of the investigation is realized, and an offer x to the defendant. A strategy for the defendant $\sigma^D : h_n^D \times x_n \mapsto \{\textit{accept, reject}\}$ maps the defendant's history h_n^D to a decision whether to accept or not offer x_n.

Solution concept. An equilibrium is a *perfect Bayesian equilibrium (PBE)*.

3 One-Period Benchmark

I clarify the decision of disclosing exculpatory evidence by the prosecutor using a simple version of the model with only one period of investigation before the trial. As further simplification, I assume that the prosecutor always investigates (it is not a decision in this case).[12] As the trial is at time T, the probability of finding new evidence is $1 - q$.

Define:

$$\tilde{\theta} = 1 - \frac{d(1-q)}{c}$$

[11] This is without loss of generality. Given that the prosecutor makes the offer, including a cost for the defendant does not affect the results as the prosecutor would incorporate the defendant's cost in her offer.

[12] This simplification does not change the main insights; it is only to simplify the analysis.

Proposition 1 *After the investigation, there is an equilibrium in which if the prosecutor gets evidence $y = h$, she discloses it, and if the prosecutor gets evidence $y = e$, she discloses it if $\theta > \tilde{\theta}$ and conceals it if $\theta \leq \tilde{\theta}$.*

The intuition of this result is given by the optimal offer that the prosecutor will make depending on the disclosure decision. First, the prosecutor always discloses $y = h$ because it induces the guilty defendant to accept the offer $x = h$. The defendant accepts $x = h$ because he would receive the same punishment at the trial if he rejects it.

Second, if the prosecutor gets exculpatory evidence, she would like to hide the evidence and make an offer that satisfies two conditions: it does not reveal the prosecutor's evidence and is accepted (otherwise, the prosecutor faces a negative payoff at the trial). The following analysis shows that when the prosecutor's prior belief about the defendant being guilty is high enough, it is impossible to satisfy these two conditions. Then the e-type prosecutor reveals the evidence.

Consider, by contradiction, that the prosecutor does not disclose exculpatory evidence for any θ. The investigation and the nondisclosure of evidence induce second-order belief uncertainty on the innocent defendant because the innocent defendant does not know whether the prosecutor knows his type. The d-type prosecutor believes the defendant is guilty with probability θ, and the e-type prosecutor knows that the defendant is innocent. Assuming no-disclosure of exculpatory evidence, the innocent defendant's belief about the prosecutor's type is the following:

$$\begin{aligned} P^I(d\text{-type prosecutor} \mid \text{no-disclosure }) = &\quad q \\ P^I(e\text{-type prosecutor} \mid \text{no-disclosure }) = &\quad 1 - q \end{aligned}$$

The guilty defendant's belief the about prosecutor's type is

$$P^G(d\text{-type prosecutor} \mid \text{no-disclosure }) = 1.$$

Second-order beliefs affect the expected punishment at trial. If there is no disclosure of evidence, the expected punishment at trial for the innocent defendant, given these beliefs, is $dq + 0(1 - q)$. The guilty defendant knows for sure that the prosecutor is d-type if there is no disclosure because the prosecutor always discloses $y = h$; therefore, his expected punishment at trial is d.

If the prosecutor is d-type, she cannot extract the full surplus from the defendant because of the second-order belief uncertainty. The innocent defendant will not accept an offer higher than $x = dq$, while the guilty defendant will accept any offer lower or equal to $x = d$. The prosecutor prefers to make an offer $x = dq$ that both defendant's type accept if the prior belief of being guilty is low, i.e., $\theta \leq 1 - \frac{d(1-q)}{c}$, while she prefers to offer $x = d$ that only the guilty defendant accepts if $\theta > 1 - \frac{d(1-q)}{c}$.

Suppose now the prosecutor is e-type. The prosecutor is able to hide the exculpatory evidence if she makes the same offer as the d-type prosecutor. It is because the innocent defendant does not know the evidence that the prosecutor has, and if the d-type and the e-type make the same offer, the defendant cannot extract information from it.

It is optimal for the e-type prosecutor to make the same offer as the d-type if $\theta \leq \tilde{\theta}$ because the innocent defendant accepts it. However, it is not optimal to make the same offer as the d-type if $\theta > \tilde{\theta}$ because the innocent defendant rejects $x = d$ and the prosecutor gets a negative payoff at trial.[13] Hence, if $\theta > \tilde{\theta}$, the e-type prosecutor must make a lower offer than the d-type prosecutor, and this lower offer reveals her private information.

In equilibrium, the prosecutor discloses $y = e$ if $\theta > \tilde{\theta}$ and offers $x = 0$, because any other offer will be rejected by the innocent defendant. The intuitive reason is that any offer $x < d$ is not sequentially rational for the d-type prosecutor. Therefore, if the innocent defendant receives an offer $x < d$, he updates his belief about the prosecutor's type to e-type with probability one.

The other equilibrium is the prosecutor does not discloses $y = e$ and directly offers $x = 0$. In terms of payoff and the main insight, those two equilibria are the same; if there is no disclosure and just an offer $x = 0$, the innocent defendant knows that with probability 1, the prosecutor has exculpatory evidence. The offer $x = 0$ is a perfect signal of $y = e$.

Offers. In equilibrium, the prosecutor offers $x = h$ if $y = h$ and the defendant accepts it. If $\theta \leq \tilde{\theta}$, the offer is $x = dq$ if $y \in \{e, d\}$ and both defendant types accept it. If $\theta > \tilde{\theta}$ the prosecutor offers $x = 0$ if $y = e$, and $x = d$ if $y = d$. Both defendant types accept the offer if $x = 0$, and only the guilty defendant accepts $x = d$.

Inefficiency. In equilibrium, if the defendant is innocent and the prosecutor does not find new evidence, they go to trial when $\theta > \tilde{\theta}$. Going to trial is socially inefficient because it is costly for the prosecutor. If the disclosure of evidence were mandatory, there would not be inefficiency because the defendant knows what evidence the prosecutor has. Then the prosecutor always offers the same sentence the defendant would get at the trial. I discuss this case in Section 6. The voluntary disclosure of evidence is ex ante inefficient when $\theta > \tilde{\theta}$ because with probability $q(1 - \theta)$, the prosecutor and defendant go to trial.

Commitment Effect. Second-order belief uncertainty allows the prosecutor to hide evidence for $\theta \leq \tilde{\theta}$; this benefits the e-type prosecutor. However, it has a downside for the d-type prosecutor because she gets an expected payoff lower than d in equilibrium. It generates a commitment effect for the prosecutor: If she could ex ante commit to disclosing any evidence, she would do it.

The reason is that, with voluntary disclosure of evidence, when $\theta \leq \tilde{\theta}$ she gets a payoff of dq from having default evidence, no matter the defendant's type. When $\theta > \tilde{\theta}$, she

[13] It cannot be an equilibrium that the innocent defendant accepts d because in that case, the e-type prosecutor offers d, which means that the innocent defendant's expected value at trial is dq.

gets an expected payoff of $\theta d + (1-\theta)(d-c)$. In both cases, she cannot extract the full surplus from the default evidence. This negative effect of the voluntary disclosure case outweighs the benefit getting a payoff of dq if $y = e$ and $\theta \leq \tilde{\theta}$. Therefore, for any θ, the prosecutor is ex ante better off if she can commit to disclosing any evidence she receives.

4 Multi-period Model

The main difference between the case with only one period and the multi-period setting is that the prosecutor can learn about the defendant's type through rejected offers in the latter case. In this Section, I show that the same intuition regarding the disclosure of exculpatory evidence remains in the presence of belief updating.

I show that the prosecutor updates her belief through rejected offers only for a specific range of prior beliefs. This updating allows her to hide evidence for a larger set of prior beliefs values than the one-period-case benchmark. I also show that the prosecutor prefers to reach an agreement in the first period for low values of θ.

4.1 Multiplicity of Equilibria

This game admits multiple equilibria. There are two qualitatively different equilibria for low values of the prior belief: The prosecutor and defendant reach an agreement during the first period, or they reach an agreement at the end of the game. Intuitively, the prosecutor prefers not to engage in an investigation when the prior belief is low enough. To do that, the prosecutor and defendant can reach an agreement the first period after investigating only the first period or reaching an agreement the last period without investigation in any period. In the main part of the paper, for low values of the prior belief, I focus on the case the prosecutor and defendant reach an agreement in the first period, as it is the best equilibria for the prosecutor when the number of periods is higher than a threshold.[14]

There are two equilibria for higher values of the prior belief (when the prosecutor prefers to investigate). The first one is when the prosecutor uses the rejected offers to update her belief to increase her expected payoff. In the second one, the prosecutor never updates her prior belief to increase her expected payoff. I focus on the first class of equilibria because that is the best equilibria for the prosecutor.

Further, given there is no discount factor, there are multiple equilibria regarding when to reach an agreement. I focus on equilibria in which the prosecutor and defendant reach an agreement at time n if they are indifferent between doing so or moving to the next period.[15] Also, if the prosecutor gets evidence $y = e$, she is indifferent between disclosing

[14] If the number of periods is lower than the referred threshold, it is unclear which equilibria provide the prosecutor with a higher payoff (Lemma 2). In Appendix A.1, I discuss the equilibrium in which the prosecutor and defendant reach an agreement at the end of the game.

[15] This selection is to rule out equilibria in which prosecutor and defendant delay reaching an agreement

it and offering $x = 0$ and not disclosing it and offering $x = 0$. Both strategies are payoff equivalent and qualitatively the same. I consider that the prosecutor discloses the evidence.[16]

In summary, I will focus on the equilibrium in which the prosecutor and defendant reach an agreement the first period for low values of θ, and they do not delay an agreement if doing so does not increase their payoffs for higher values of θ. Also, in the equilibrium that the prosecutor updates her belief only when it increases her expected payoff. In Appendix A.1, I analyze the case where the prosecutor might not reach an agreement in the first period, and no investigation is sustained as equilibrium.

4.2 Agreement the first period for low values of prior belief

An important observation is, given that the investigation induces second-order belief uncertainty and results in the d-type prosecutor getting a lower payoff, the prosecutor might prefer not to investigate for low values of θ.

Define:

$$\underline{\theta}^N = \frac{dq^{\frac{1}{N}}}{h - dq}.$$

Lemma 1 *Assume $\theta \leq \underline{\theta}^N$. In equilibrium, the prosecutor investigates in the first period. If $y = h$, she discloses it, offers $x = h$, and the guilty defendant accepts the offer. If $y \in \{e, d\}$ she does not disclose it, offers $x = dq^{\frac{1}{N}}$, and both defendant types accept.*

Lemma 1 says the prosecutor and the defendant reach an agreement the first period for $\theta \leq \underline{\theta}^N$. The intuition is that the prosecutor prefers to make an offer that is accepted because if the probability that the defendant is guilty is low enough, the prosecutor is better off not investigating for new evidence the following period because of the risk of finding $y = e$ is higher than finding $y = h$.

If the prosecutor prefers not to investigate the following periods, the way to sustain no-investigation in equilibrium is removing the incentive to deviate to investigate in the subsequent periods. To do that, the prosecutor needs to separate the guilty from the innocent by making an offer the guilty defendant strictly prefers to accept. The highest offer the guilty defendant accepts for sure is the innocent defendant's continuation punishment if there is not going to be more investigation.[17]

with no change in information and payoffs. It can be interpreted as a weak form of players being impatient.

[16]this selection only affects $y = e$. If the prosecutor gets evidence $y = h$, in any equilibrium the prosecutor discloses it before offering $x = h$.

[17]This is because if this offer is rejected, the prosecutor does not investigate in the following periods since only the innocent defendant rejects it. Therefore, the innocent defendant is indifferent between accepting it and rejecting it. If the offer is higher than the innocent defendant's continuation punishment, the innocent defendant rejects it for sure, and the prosecutor would make a lower offer next period (to avoid trial). Hence, the guilty defendant also rejects it because there will be a lower offer next period.

The probability that the defendant assigns to the prosecutor being a d-type decreases as the prosecutor investigates because of the probability of finding $y = e$. Hence, after one period of investigation, the innocent defendant's continuation punishment is $dq^{\frac{1}{N}}$.

4.3 Disclosure Decision for higher prior belief value

For clearer exposition in the rest of the Section, I define $\theta^{(n)}$ as the prosecutor's belief about the defendant's being guilty at the beginning of period n, where $\theta = \theta^{(n=1)}$ is the prior belief.

An important observation for the results in this Section is the relevance of the last period. The decisions of investigating, disclosing evidence, and making an offer depend on the decision of disclosing or hiding evidence in the last period. Since the probability of finding new evidence at each period is $1 - q^{\frac{1}{N}}$, the total probability of finding new evidence by the end of the plea bargaining phase, if the prosecutor investigates during all periods, is $1 - q$.

The main result of this Section generalizes the result that the prosecutor discloses exculpatory evidence for high values of the prior belief. The cutoff value of the prior belief such that the prosecutor discloses exculpatory evidence is increasing in the number of periods but bounded by a limit value lower than 1.

Define:

$$\bar{\theta}^N = \frac{c - d(1 - q^{\frac{1}{N}})}{c + dq^{\frac{1}{N}}(1 - q^{\frac{N-1}{N}})}$$

Proposition 2 *For $\theta > \underline{\theta}^N$, the prosecutor investigates at every period as long as agreement is not reached.*
1. The prosecutor discloses $y = h$ as soon as she gets it, and an agreement is reached the same period with an offer $x = h$.
2. The prosecutor discloses $y = e$ as soon as she gets it for $\theta > \bar{\theta}^N$, and an agreement is reached with an offer $x = 0$ the same period. The prosecutor does not disclose $y = e$ if $\theta \in (\underline{\theta}^N, \bar{\theta}^N]$, and an agreement is reached with an offer $x = dq$ at $n = N$.

To explain the intuition of the result in Proposition 2, I separate the analysis into two cases:

1. No disclosure for $\theta \in (\underline{\theta}^N, \bar{\theta}^N]$: The description of the equilibrium that sustain no disclosure is further separated in two cases

- $\theta \in (\underline{\theta}^N, \tilde{\theta}]$: If $y = h$ they reach an agreement as explained in Proposition 2. If $y \in \{e, d\}$, at $n < N$ the prosecutor makes offers that are rejected for sure by the defendant, and at $n = N$ she offers $x = dq$ that is accepted by both types of the defendant.

- $\theta \in (\tilde{\theta}, \bar{\theta}^N]$: If $y = h$ they reach an agreement as explained in Proposition 2. If $y \in \{e, d\}$, at the end of $n = 1$ the prosecutor offers $x = (1 - q^{\frac{N-1}{N}})h + q^{\frac{N-1}{N}} dq$, which is accepted with probability $\mu^G = \frac{\theta - \tilde{\theta}}{\theta(1-\tilde{\theta})}$ by the guilty defendant and rejected by the innocent defendant. Prosecutor updates her prior belief to $\theta^{(n=2)} = \tilde{\theta}$. Then, for $2 \leq n < N$ she makes offers that are rejected by the defendant, and at $n = N$ she offers $x = dq$, which is accepted by the defendant.

The prosecutor is able to hide exculpatory evidence if $\theta \leq \tilde{\theta}$ because she investigates every period as long as she does not get evidence $y = h$. It implies that at the end of the last period, the probability that she has $y = d$ for the defendant is q. Hence, applying Proposition 1 she is able to hide exculpatory evidence if $\theta^{(n=N)} \leq \tilde{\theta}$. Furthermore, for these prior belief values, the prosecutor does not update her belief through rejected offers. That means that she makes offers $x > h$ such that both defendant types reject for sure, and therefore the rejection does not bring any new information. In conclusion, in equilibrium, the posterior belief when they reach an agreement is the same that the prior belief $\theta^{(n=N)} = \theta$ if the prosecutor does not receive any information by time N.

For $N > 1$, the range of prior belief values that allows the prosecutor to hide exculpatory evidence in equilibrium is expanded, including the values $\theta \in (\tilde{\theta}, \bar{\theta}^N]$. It is because the prosecutor makes an offer at the end of the first period of investigation that the guilty defendant is indifferent to accept, and therefore accepts with probability $\mu^G = \frac{\theta - \tilde{\theta}}{\theta(1-\tilde{\theta})}$, and the innocent defendant rejects it for sure. The guilty defendant is indifferent between accepting and rejecting the offer $x = (1 - q^{\frac{N-1}{N}})h + q^{\frac{N-1}{N}} dq$ because the offer is equal to her continuation payoff if she rejects the offer and the prosecutor updated her belief. Under this strategy, the prosecutor updates her belief to $\theta^{(n=2)} = \tilde{\theta}$ if there is a rejection.

Note that the prosecutor updates her belief following a rejected offer only in the first period. In the following periods this does not happen because she is able to hide exculpatory evidence (since $\theta^{(n=2)} = \tilde{\theta}$). Finally, note that in order to hide exculpatory evidence, the prosecutor needs to wait until the last period to offer $x = dq$ following what the d-prosecutor would do.

If $y \in \{e, d\}$, the prosecutor does not increase her payoff by making intermediate offers that the defendant might accept (with the exception of the offer the first period if $\theta \in (\tilde{\theta}, \bar{\theta}^N]$ as explained above).

An intermediate offer that is accepted by both defendant types has to be at most the expected punishment of the innocent defendant. That is, at most $dq^{\frac{n^*}{N}}$ if the offer is at period n^*, which is lower than making an offer that is rejected for sure and play the continuation game. It is not possible for the prosecutor to make an offer that only the innocent defendant accepts because the guilty defendant will always accept it as it has to be lower than his expected punishment.

It is payoff equivalent for the prosecutor to make an offer that the innocent defendant rejects and the guilty defendant accepts with some probability, and to make an offer that both defendant types reject for sure.[18] That intermediary offer that the guilty defendant accepts with some probability has to be equal to the guilty defendant's continuation punishment. The offer and the probability of acceptance by the guilty defendant have to be such that the prosecutor updates her belief to $\theta' \in (\underline{\theta}^N, \theta)$ following a rejection—if the prosecutor updates her prior belief to $\theta' \leq \underline{\theta}^N$, the guilty defendant rejects the offer for sure given that next period the prosecutor will make a lower offer. Therefore the continuation punishments do not change, and the prosecutor gets the same expected payoff.

2. Disclosure for $\theta \in (\bar{\theta}^N, 1]$: Prosecutor discloses $y = e$ and $y = d$ as soon as she gets it, and they reach an agreement that period. If $y = d$ she continues investigating, and she offers $x = d$ in the last period that is accepted by the guilty defendant and rejected by the innocent defendant. While $y = d$ and $n < N$, she makes offers that are rejected for sure by the defendant.

In this case, the prosecutor also does not increase her payoff by making an intermediate offer that is accepted with some probability if $y = d$. As before, making an offer that is accepted by both defendant types has to be equal to the innocent defendant's continuation value, which is not optimal for the prosecutor. And making an offer that the innocent defendant rejects and the guilty defendant accepts with some probability is payoff equivalent to an offer that both defendant types reject. The former type of offer has to be equal to the guilty defendant's continuation value and can only update the prosecutor's belief to $\theta' \in (\bar{\theta}^N, \theta)$. Therefore, it is payoff equivalent to an offer that both defendants reject and does not generate extra information.

The first part of proposition 2 says that the prosecutor prefers to investigate every period as long as they have not reached an agreement. For the ranges of prior belief values in which she does hide evidence, a deviation to stop investigating is not profitable because the innocent defendant cannot differentiate between the prosecutor not investigating and not disclosing. Therefore she cannot make a higher offer, and she forgoes the option of finding h.

For the range of prior belief values in which the prosecutor discloses exculpatory evidence, if she decides not to investigate anymore, she gets a payoff of d if the defendant is guilty and $d - c$ if the defendant is innocent. On the other hand, if she continues investigating, she has an expected continuation payoff of $(1 - q^*)h + q^*d$ if the defendant is guilty, and $q^*(d - c)$ if the defendant is innocent, where q^* is the remaining probability of finding new evidence. When the prior belief is high enough, the lower expected payoff when the defendant is innocent if she continues investigating is lower than the gain if the defendant is guilty. Hence, the prosecutor always continues investigating.

[18] It can not be an offer that the innocent defendant rejects, and the guilty defendant accepts for sure because in that case, a rejection induces a posterior belief of $\theta' = 0$; therefore the guilty defendant rejects the offer.

4.4 Efficiency

The prosecutor and the defendant fail to reach an agreement if $\theta > \bar{\theta}^N$ and $y = d$ because the innocent defendant does not accept the offer the prosecutor makes. In this case, the prosecutor goes to trial if the defendant is innocent. Formally, the probability of going to trial is

$$P(trial) = \begin{cases} 0 & \text{if } \theta \leq \bar{\theta}^N \\ q(1-\theta) & \text{if } \theta > \bar{\theta}^N. \end{cases}$$

Note that $\underline{\theta}^N$ and $\bar{\theta}^N$ increase with N. Hence the range of values in which the prosecutor discloses exculpatory evidence is smaller when the negotiation period is divided into more periods, and the range of values in which the prosecutor and defendant reach an agreement the first period for sure is larger. In other words, the equilibrium outcome is less inefficient when N increases.

4.5 Payoffs

The prosecutor's expected payoff, for a given θ, is weakly increasing in the number of periods for $\theta < \bar{\theta}^N$. The defendant's expected punishment is also affected by the number of periods for $\theta < \bar{\theta}^N$. The innocent defendant is weakly worse off if there are more periods of investigation because the prosecutor investigates only one period, that is a smaller fraction of the plea bargaining phase if θ is low, and therefore less likely to get $y = e$. In contrast, the guilty defendant benefits from less investigation because it is less likely to find $y = h$. Prosecutor and defendant are indifferent to the number of period for $\theta \geq \bar{\theta}^N$.

Corollary 1 *The prosecutor's expected payoff and the innocent defendant's expected punishment is maximized at $N \to \infty$. The guilty defendant's expected punishment is minimized at $N \to \infty$.*

As an example, Figure 2 compares the cases when the length T of the plea bargaining phase is divided into one, two, and infinite periods (i.e., $N \to \infty$). For higher N, the prosecutor is better off for low values of θ because, after investigating in the first period, she can make an offer that both defendant types accept, and this offer is higher if there are more periods. The prosecutor is also better off with more periods if θ belongs to the interval $(\tilde{\theta}, \bar{\theta}]$ because she can make an intermediate offer that increases the θ threshold in which she can hide evidence and transfer a higher punishment from the guilty defendant to the innocent defendant.

The innocent defendant is worse off for higher N because the range of θ in which the prosecutor only investigates one period is larger. Also, in that interval, the offer the prosecutor makes is higher if N is higher. In the limit case $N \to \infty$, the prosecutor offers $x = d$ because the investigation in the first period is negligible compared with the rest of the periods.

The guilty defendant is better off with less investigation. He is also better off with more periods because the range of θ values in which the prosecutor discloses exculpatory evidence is smaller. It affects the guilty defendant because if the prosecutor hides exculpatory evidence, she makes a lower effort when $y = d$ than when she discloses it.

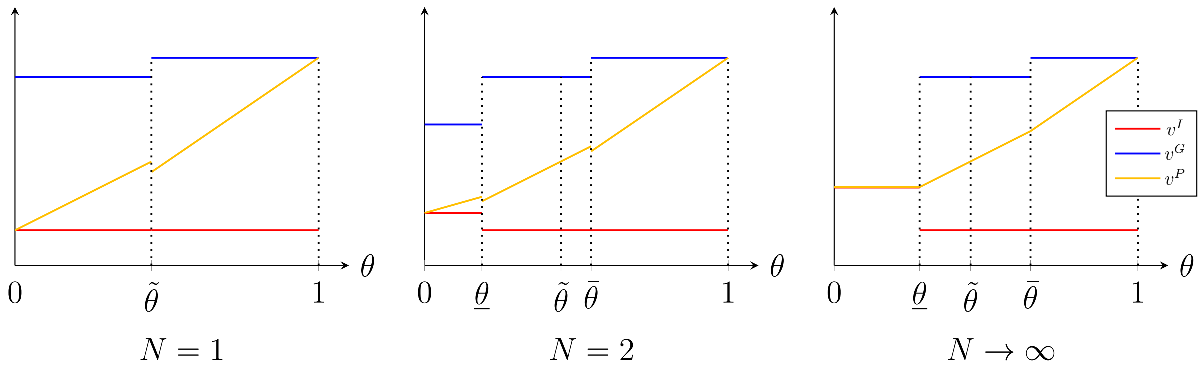

Figure 2: Expected payoffs and punishments comparison between $N = 1$, $N = 2$, and $N \to \infty$.

5 Frequent-offer Limit

In this Section I consider the frequent-offer limit case, where $N \to \infty$ keeping T constant. The implication is that the probability of finding new evidence at each period is arbitrarily low because $\Delta \to 0$.

The analysis of this case is important for two reasons. First, the high-frequency limit provides intuition on a continuous investigation by the prosecutor, whereby she can interrupt the investigation to make an offer at any point. Second, given this is the case that maximizes the prosecutor expected payoffs, it provides a good framework to compare the cases of voluntary disclosure and mandatory disclosure of information in the next Section, without conditioning in the number of periods N.

By definition, the limit values of the θ cutoffs when $N \to \infty$ are

$$\lim_{N\to\infty} \underline{\theta}^N = \frac{d}{h - dq} \equiv \underline{\theta}, \text{ and } \quad \lim_{N\to\infty} \bar{\theta}^N = \frac{c}{c + d(1-q)} \equiv \bar{\theta}.$$

The prosecutor ends the game at $t = 0$ for $\theta \leq \underline{\theta}$. For $\theta > \underline{\theta}$, the time the game ends depends on the evidence and the value of θ because the prosecutor uses different strategies depending on θ. In this Section I show that the game has a deadline effect—the probability of ending the game has a mass point at the deadline T.

5.1 The Path of Agreements.

Deadline effects in pretrial negotiation have been studied in Spier (1992); Ma and Manove (1993); Fuchs and Skrzypacz (2013); and Vasserman and Yildiz (2019). They have also been observed in experimental studies by Roth et al. (1988) and Güth et al. (2005).[19]

Proposition 3 *There is a deadline effect: The probability of reaching an agreement has a mass point at T. The game ends by T with probability 1 for $\theta \in (\underline{\theta}, \bar{\theta}]$ for both defendant types, and ends by T with probability 1 for $\theta \in (\bar{\theta}, 1]$ if the defendant is guilty.*

If $\theta \in (\underline{\theta}, \bar{\theta}]$, the prosecutor hides exculpatory evidence; therefore if the defendant is innocent the game ends at time T. Hence the prosecutor ends the game at the deadline. If the defendant is guilty, she ends the game as soon as she gets evidence $y = h$ or at time T if she never gets new evidence; therefore, there is also a deadline effect. Note that if $\theta \in (q, \bar{\theta}]$, the prosecutor makes an initial offer that is rejected by the innocent defendant and accepted by the guilty defendant with probability μ^G. Let τ be the time by which the game ends. The probability that τ is less than t when the defendant is guilty is given by:

If $\theta \in (\underline{\theta}, \tilde{\theta}]$, then

$$P_G(\tau \leq t) = \begin{cases} 1 - e^{-\lambda \frac{t}{T}} & \text{if } t < T \\ 1 & \text{if } t = T. \end{cases}$$

If $\theta \in (\tilde{\theta}, \bar{\theta}]$, then

$$P_G(\tau \leq t) = \begin{cases} \mu^G & \text{if } t = 0 \\ 1 - e^{-\lambda \frac{t}{T}}(1 - \mu^G) & \text{if } t \in (0, T) \\ 1 & \text{if } t = T. \end{cases}$$

And for the innocent defendant for $\theta \in (\underline{\theta}, \bar{\theta}]$:

$$P_I(\tau \leq t) = \begin{cases} 0 & \text{if } t < T \\ 1 & \text{if } t = T. \end{cases}$$

For $\theta > \bar{\theta}$, the prosecutor reveals any new evidence. Nevertheless, if she does not get new evidence the game ends at T only if the defendant is guilty; if the defendant is innocent, they go to trial. This means that there is a deadline effect only when the defendant is guilty. The probability that the game ends by time t when the defendant is

[19] Other papers that find a deadline effect are Cramton and Tracy (1992); and Fershtman and Seidmann (1993).

guilty for $\theta > \bar{\theta}$ is

$$P_G(\tau \leq t) = \begin{cases} 1 - e^{-\lambda \frac{t}{T}} & \text{if } t < T \\ 1 & \text{if } t = T. \end{cases}$$

And for the innocent defendant for $\theta > \bar{\theta}$:

$$P_I(\tau \leq t) = 1 - e^{-\lambda \frac{t}{T}} \quad \text{for } t \leq T.$$

I consider the trial is at period $T+1$. If the probability of ending the game at or before T is less than one, the game ends at trial at time $T+1$ with the remaining probability.

Figure 3 illustrates the probability of ending the game by time t depending on the defendant's type and the prior belief.

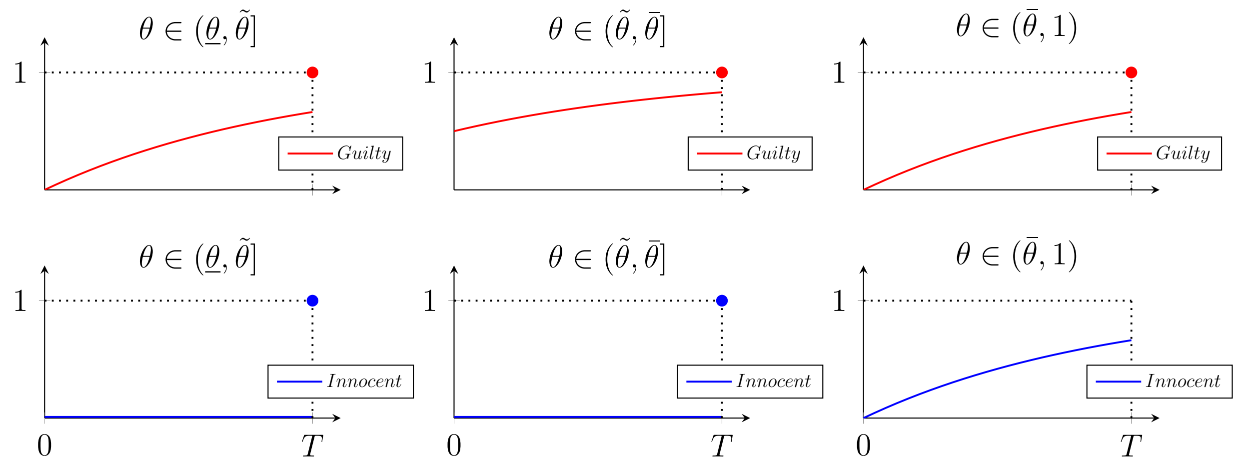

Figure 3: Probability of ending the game by time $t \leq T$.
Values: $\mu = 0.5$, $\lambda = 0.8$, $T = 30$, $q = 0.55$.
Left: $\theta \in (\underline{\theta}, \tilde{\theta}]$, middle: $\theta \in (\tilde{\theta}, \bar{\theta}]$, right: $\theta \in (\bar{\theta}, 1]$.

6 Mandatory Disclosure of Evidence

The Brady Rule is the legal requirement that the prosecutor must disclose all evidence she has—default, incriminating or exculpatory—to the defendant at trial. The Brady Rule is not always extended to pretrial negotiations; the Fifth Circuit court recently[20] joined the First, Second, and Fourth Circuits by ruling that criminal defendants are not constitutionally entitled to exculpatory evidence prior to entering a guilty plea.[21] The Seventh, Ninth, and Tenth Circuits have ruled that exculpatory evidence must be disclosed before entering a guilty plea. The United States Supreme Court has not ruled

[20] In 2018, in deciding *Alvarez v. City of Brownsville*.
[21] See Petegorsky (2012); Grossman (2016); and Casey (2020).

on the issue.[22]

In this Section, I assume the Brady Rule applies to the pretrial negotiation process as well as the trial. I compare the equilibrium under Brady Rule (mandatory disclosure of evidence during plea bargaining) and voluntary disclosure of evidence. I suggest that the Brady Rule should be extended to pretrial negotiations because it improves efficiency. I also show that the Brady Rule case's outcomes are closer to assigning a high punishment to the defendant if he is guilty and set the defendant free if he is innocent. In the following, I refer to the mandatory disclosure of evidence during plea bargaining as the Brady Rule case.

6.1 The Brady Rule: Mandatory Disclosure

The prosecutor does not induce second-order belief uncertainty in the defendant when she investigates because the defendant knows the evidence the prosecutor has before any offer. Therefore, the prosecutor ends the game if she gets evidence $y = e$ or $y = h$ by offering $x = 0$ and $x = h$, respectively. The trade-off the d-type prosecutor faces each period is whether to investigate.

Define:

$$\underline{\theta}^{BR} = \frac{d}{h}.$$

Proposition 4 *The following results hold:*

(i) For $\theta \leq \underline{\theta}^{BR}$, the prosecutor does not investigate in the first period. She offers $x = d$ at the end of the first period, and the defendant accepts.

(ii) For $\theta > \underline{\theta}^{BR}$, the prosecutor investigates in every period, as long as the game has not ended. If she gets $y = e$ or $y = h$, she offers $x = 0$ and $x = h$, respectively and the defendant accepts the offer. If she does not get new evidence at $n < N$, she offers $x = h$, which is rejected for sure by both defendant types. If she does not get new evidence at $n = N$, she offers $x = d$, which is accepted for sure by both defendant types.

Proposition 4 says that a d-type prosecutor either investigates every period or never investigates. The reason is that for low values of θ, the risk of finding exculpatory evidence is higher than the benefit of finding $y = h$. The opposite is true for high values of θ. Note further that the cutoff and the equilibrium do not depend on the number of periods N.

It also says that the d-type prosecutor makes an offer $x = d$ at $n = N$ that the defendant accepts. Hence, the prosecutor and the defendant always reach an agreement in the plea bargaining phase with Brady Rule, which implies that there are no inefficiencies

[22]See Casey (2020).

related to going to trial.

Corollary 2 *The equilibrium under Brady Rule is efficient: The prosecutor and the defendant never go to trial.*

Figure 4 shows the payoffs and punishments with mandatory disclosure of evidence.

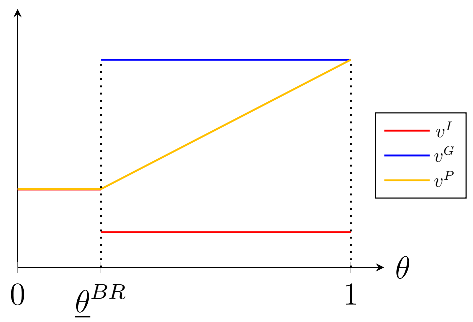

Figure 4: Expected payoff and punishments with mandatory disclosure of evidence.

6.2 The Path of Agreements

In the Brady Rule case, there also is a deadline effect. However, in this case, it is the same for both the guilty and the innocent type. The prosecutor and the defendant reach an agreement at the first period when $\theta \leq \underline{\theta}^{BR}$, and an agreement either as soon as the prosecutor gets new evidence or at the deadline if she does not get new evidence if $\theta > \underline{\theta}^{BR}$.

Considering the limit-offer case (i.e., $N \to \infty$), let τ^{BR} denote the time at which the prosecutor and defendant reach an agreement. The probability that the game ends by t when $\theta > \underline{\theta}^{BR}$ is given by

$$P\big(\tau^{BR} \leq t\big) = \begin{cases} 1 - e^{-\lambda \frac{t}{T}} & \text{if } t < T \\ 1 & \text{if } t = T \end{cases}$$

Figure 5 graphically shows the path of agreements when $\theta > \underline{\theta}^{BR}$. There are three main differences between the voluntary case and the Brady Rule case. First, in the voluntary case, the probability of ending the game by t is different for the innocent and the guilty types, while in the Brady Rule case is the same for both types. Second, there is a positive probability of no agreement in the voluntary disclosure case at the plea bargaining phase, while under Brady Rule, the prosecutor and the defendant always reach an agreement. Third, in the voluntary disclosure of evidence case, the probability of ending the game at t has a mass point at $t = 0$, while under Brady Rule, that mass point does not exist.

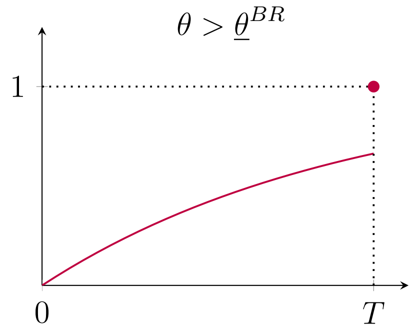

Figure 5: Probability of ending the game by time $t \leq T$ for $\theta > \underline{\theta}^{BR}$, for both defendant types. Values: $\lambda = 0.8$, $T = 30$, $q = 0.55$.

7 Voluntary v. Mandatory Disclosure

In this Section I compare the voluntary disclosure case with the mandatory disclosure case. I show three important results and policy implications: Mandatory disclosure of evidence is more efficient, the outcome under mandatory disclosure is *fairer*, and there is a commitment effect.

1. Efficiency. As shown in Section 4.4, the game is inefficient under voluntary disclosure of evidence. With a positive probability for higher values of θ, the prosecutor and defendant go to trial, which is costly for the prosecutor. It is not the case in the mandatory disclosure of evidence case (as shown in Section 6.1), as they always reach an agreement before the trial.

2. Fairness. A policy-relevant question is which system generates outcomes closer to a fair system. I define a fair system as the one who gives a punishment of 0 to the innocent defendant and punishment of h to the guilty defendant. In this Section, I show the mandatory disclosure of evidence generates outcomes closer to a fair system compared to the voluntary case, therefore is socially desirable from a normative point of view.

Proposition 5 *Comparing the frequent-offer-limit voluntary disclosure case and the mandatory case:*
1. The innocent defendant is weakly better off with mandatory disclosure of evidence.
2. The guilty defendant is weakly worse off with mandatory disclosure of evidence.

Note that the innocent defendant gets the same expected payoff when the prosecutor prefers to investigate in both disclosure cases. The difference is that the innocent type is worse off when there is no investigation because she gets a punishment of d. In the mandatory disclosure case, the lowest prior belief that induces the prosecutor to investigate is lower than in the voluntary disclosure case: $\underline{\theta}^{BR} < \underline{\theta}$. In other words, the prosecutor investigates for a larger range of priors beliefs in the mandatory case. Therefore, the innocent defendant is weakly better off under the Brady Rule.

The guilty defendant is better off with voluntary disclosure of evidence as in this case; there is less investigation and lower offers. The guilty defendant is weakly better off in the voluntary case when the prosecutor prefers not to investigate because the probability of finding $y = h$ is zero. The guilty defendant also prefers the voluntary case in the range of values in which the prosecutor hides exculpatory evidence because the prosecutor offers him a lower sentence when she has default evidence. Finally, he gets the same punishment in both disclosure cases when the prosecutor discloses exculpatory evidence.

Figure 6 graphically compares the mandatory disclosure case with the frequent-offer-limit voluntary case.

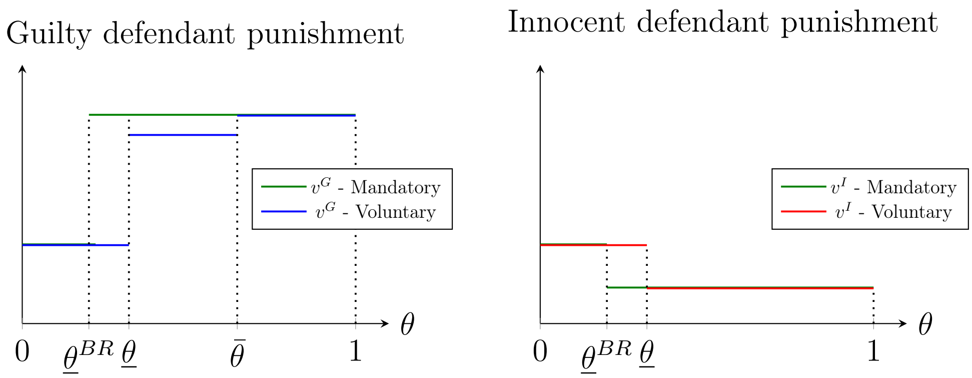

Figure 6: Comparison of punishments with voluntary disclosure and mandatory disclosure of evidence.

3. Commitment Effect. If the prosecutor can choose which disclosure case and commit to that one, which one will she choose? In this Section I show that the prosecutor prefers the mandatory disclosure case. That is, the prosecutor is better off without the ability to decide whether to disclose evidence after observing it.

Proposition 6 *The prosecutor is weakly better off with mandatory disclosure of evidence.*

The prosecutor is better off with mandatory disclosure of evidence for two reasons: (1) she extracts the full surplus from each defendant type if she investigates, and (2) she investigates for a larger range of prior beliefs.

The prosecutor extracts all the surplus under Brady Rule. When the disclosure of evidence is mandatory, the agreement reached by the prosecutor and the defendant is either h if the evidence is incriminating, d if it is default, or zero if it is exculpatory. It implies that the prosecutor gets the expected payoff after investigation from each defendant.

When the disclosure of evidence is voluntary, there are two options. First, if the prior belief is below $\bar{\theta}$, the prosecutor hides exculpatory evidence. Second, if the prior belief is above $\bar{\theta}$, she discloses exculpatory evidence.

i) If the prosecutor hides exculpatory evidence, she gets the same expected payoff if the defendant is innocent compared to the mandatory disclosure of evidence case. The prosecutor and the defendant agree on a sentence equal to dq, which is equal to the expected punishment in the mandatory case. However, the prosecutor gets a lower payoff compared to the mandatory case if the defendant is guilty. In the voluntary case, they agree on a sentence h if the evidence is incriminating or dq if the prosecutor has default evidence.

ii) If the prosecutor discloses exculpatory evidence, the prosecutor gets the same payoff from the guilty defendant in the mandatory and voluntary cases. When the prior belief is high enough, she offers d when she has default evidence, and the defendant accepts the offer. However, she gets zero payoff if the defendant is innocent because she either discloses the exculpatory evidence, resulting in a payoff of zero or her offer of d is rejected by the defendant, ending in a payoff of zero at trial.

The prosecutor investigates more under Brady Rule. In both the mandatory and voluntary disclosure cases, the prosecutor prefers not to investigate and reach an agreement at $t = 0$ for lower values of the prior belief. However, the threshold such that she prefers to investigate is lower under Brady Rule. That is, if the disclosure of evidence is mandatory, the prosecutor investigates for a larger range of θ values compared to the voluntary disclosure case.

The prosecutor decides to investigate instead of reaching an immediate agreement if that action gives her an expected payoff higher than d. This happens for lower values of θ in the Brady Rule case compared to the voluntary disclosure of evidence case because the Brady Rule case gives her a higher expected payoff as explained above.

The analysis above implies that there is a commitment effect for the prosecutor: If she could credibly commit at the beginning of the game to disclose all of her evidence, she would do it. Because she cannot commit to disclose evidence when the disclosure is voluntary, she has the incentive to hide exculpatory evidence when she gets it; however, the defendant anticipates this, impeding her ability to extract the full surplus. Therefore, she is better off with mandatory disclosure of evidence.

Figure 7 graphically compares the voluntary and mandatory disclosure cases.

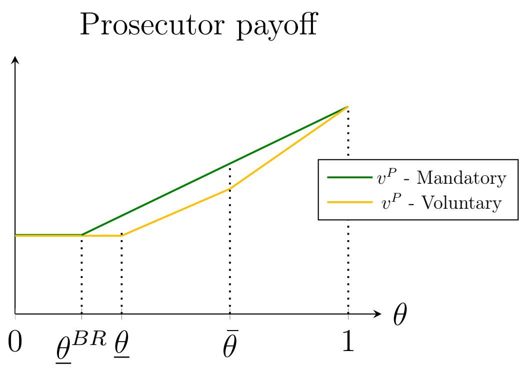

Figure 7: Comparison of payoffs with voluntary disclosure and mandatory disclosure of evidence.

8 Concluding Remarks

With voluntary disclosure of evidence in plea bargaining, in equilibrium, the prosecutor hides exculpatory evidence when the prior belief about the defendant being guilty is low. However, she discloses the exculpatory evidence when the prior belief about the defendant being guilty is high enough. It means that a prosecutor who is purely sentence-motivated may still disclose exculpatory evidence.

Nevertheless, even though there is disclosure of exculpatory evidence when disclosure is voluntary during the plea bargaining phase, the mandatory disclosure protocol during plea bargaining is, from a normative point of view, socially desirable for two reasons: It is efficient in the sense that the prosecutor and defendant always reach an agreement before trial, and because the defendant gets a higher sentence if he is guilty and a lower sentence is he is innocent. Finally, I showed that the prosecutor prefers the mandatory disclosure case.

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Appendices

A Extensions

A.1 No-investigation Equilibria for low prior belief

In the main part of the paper, I described the equilibrium in which there is always investigation in the first period, and the prosecutor and defendant reach an agreement at the end of it. In this Section, I show the class of equilibria in which the prosecutor does not investigate at any period for low values of θ. I call *immediate-agreement equilibrium* to the former and *no-investigation equilibrium* to the latter. These results do not change the main findings of the paper.

The no-investigation equilibrium cannot be supported as equilibrium for high values of θ because the incentives to investigate to get $y = h$ are increasing with θ. If θ is low, the prosecutor might prefer not to investigate if the innocent defendant accepts $x = d$ with some positive probability.

Note that if the innocent defendant accepts $x = d$ with probability 1, the prosecutor will deviate to investigate because she benefits from finding $y = h$. She is not affected by finding $y = e$ because she can offer $y = d$, and the defendant will accept. A no-investigation equilibrium can exist because the innocent defendant accepts an offer $x \leq d$ with a probability lower than 1. It induces the prosecutor to not deviate because of the possibility of getting $y = e$ and getting a negative payoff at the trial.

I define $\mu_n^I(d)$ as the probability that the innocent defendant accepts $x = d$ at period n. For each sequence $\{\mu_1^I(d), \mu_2^I(d), \ldots, \mu_{N-1}^I(d), \mu_N^I(d)\}$ of probabilities of accepting $x = d$ at each period n, I define:

$$\utilde{\theta}^N = \begin{cases} \frac{d\tilde{\mu}^I_{1,..,N}(d)}{h-d\left(1-\tilde{\mu}^I_{1,..,N}(d)\right)} & \text{if } \mu_N^I(d) \in \left[0, \frac{c}{d+c}\right) \\ \frac{\left(1-\mu^I_{1,..,N}(d)\right)c}{h-d+\left(1-\mu^I_{1,..,N}(d)\right)c} & \text{if } \mu_N^I(d) \in \left[\frac{c}{d+c}, 1\right], \end{cases}$$

where $\tilde{\mu}^I_{1,..,N}(d) = \mu_N^I(d) \prod_{j=1}^{N-1} \left(1 - \mu_j^I(d)\right)$ is the probability that the innocent defendant accepts $x = d$ at $n = N$, and $\mu^I_{1,..,N}(d) = 1 - \prod_{j=1}^{N} \left(1 - \mu_j^I(d)\right)$ is the probability that the innocent defendant accepts $x = d$ between $n = 1$ and $n = N$.

Proposition 7 *For $\theta \leq \utilde{\theta}^N$, the prosecutor never investigates and offers $x = d$ at the end of each period. The guilty defendant accepts it the first period, and the game ends, and the innocent defendant accepts it with probability $\mu_n^I(d)$ at each period n.*

The guilty defendant does not deviate because $x = d$ is the best offer he can receive. The innocent defendant does not deviate because he is indifferent between accepting d or getting d at the trial. The prosecutor does not make another offer because it will be rejected for sure, and she does not deviate to investigate because her expected continuation payoff for the deviation is lower than no-investigation, given the possibility of finding $y = e$.

Note that $\utilde{\theta}^N = 0$ if $\mu_n^I(d) = 1$ for any n. The intuition is that if the innocent defendant accepts $x = d$ for sure at some period, the prosecutor deviates to investigate. For $\theta > \utilde{\theta}^N$, the equilibrium is the same than the immediate-agreement equilibrium described in the main part of the paper. Note that $\utilde{\theta}^N < \underline{\theta}^N$ for each N.

The innocent defendant is worse off for $\theta \leq \utilde{\theta}^N$ compared to the immediate-agreement equilibrium payoff because, in the no-investigation equilibrium, he gets his highest possible punishment if θ is small enough. The guilty defendant is better off in the no-investigation equilibrium because he only gets d as punishment. It is unclear whether the prosecutor is better or worse off in the no-investigation equilibrium than the immediate-agreement equilibrium. The trade-off for $\theta \leq \utilde{\theta}$ is:

1. If there is an investigation in the first period, the prosecutor can find evidence

$y = h$. If not, she offers $x = q^{\frac{1}{N}}d$, which is lower than d, but it is accepted with probability one by both defendant types.

2. If there is no investigation in every period, the prosecutor offers $x = d$, but the innocent defendant accepts it with a probability lower than 1.

If N is low, investigating the first period is costly for the prosecutor (in the sense of reducing her expected payoff); the extreme case is $N = 1$, in which the offer the prosecutor makes is reduced to $x = dq$. Nevertheless, if N is large, investigating only one period generates a slight decrease in the prosecutor's offer. Intuitively, the prosecutor will be better off in the investigation equilibrium when N is large. This intuition is captured by Lemma 2.

Lemma 2 *Consider a sequence of probability of acceptance $\mu^I_n(d)$ for all n. For $\theta \leq \underset{\sim}{\theta}^N$ there is a N^* such that for $N \geq N^*$ the prosecutor is better off in the immediate-agreement equilibrium than in the no-investigation equilibrium.*

Figures 8 and 9 illustrate how payoffs change depending on $\mu^I(d)$ and N.

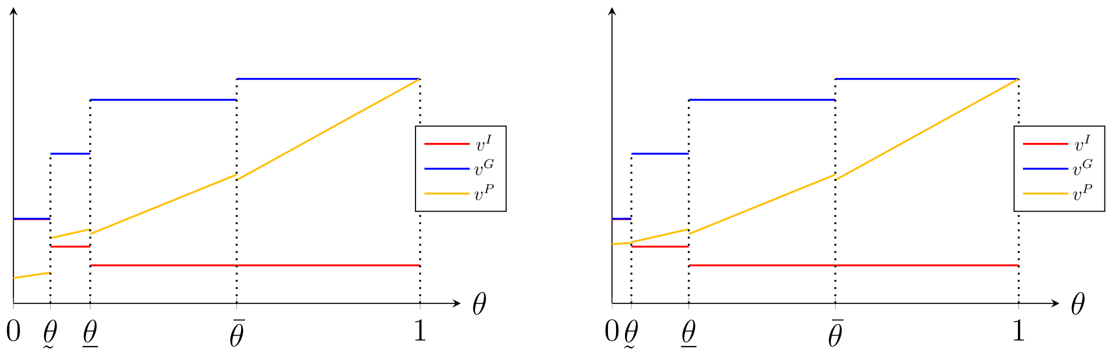

Figure 8: Expected payoffs and punishments with no-investigation equilibrium and $N = 2$. The left panel corresponds to $\mu^d = 0.3$ and the right panel to $\mu^d = 0.7$.

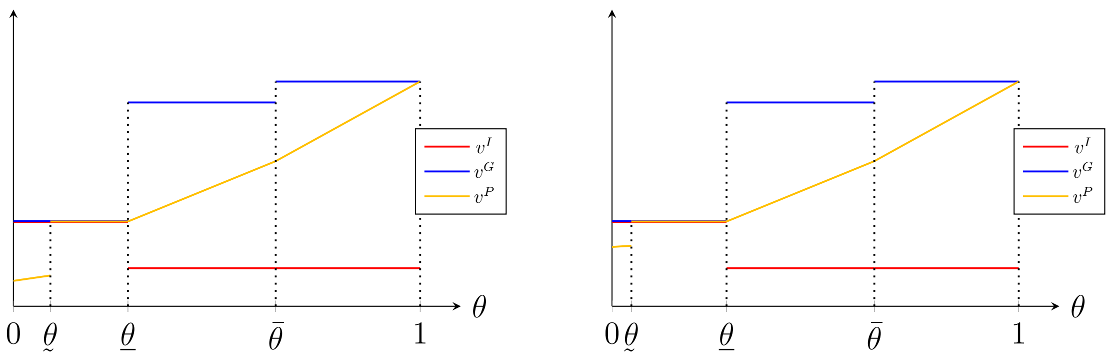

Figure 9: Expected payoffs and punishments with no-investigation equilibrium and $N \to \infty$. The left panel corresponds to $\mu^d = 0.3$ and the right panel to $\mu^d = 0.7$.

A.2 Public Investigation

This extension shows that if the investigation decision is public information, the prosecutor does not investigate for low values of θ. The equilibrium with public and private investigation decisions coincides when $N \to \infty$ and T is fixed.

If the prosecutor decides not to investigate, she does not induce second-order belief uncertainty in the defendant; he knows the prosecutor has evidence $y = d$.

Define:

$$\underline{\theta}^{Public} = \frac{d}{h - dq}.$$

Lemma 3 *If $\theta \leq \underline{\theta}^{Public}$ the prosecutor does not investigate, and she offers $x = d$ at the end of period $n = 1$. The defendant accepts it, and the game ends at the first period.*

First, note that as in the case of private investigation, the defendant's continuation punishment is decreasing in the number of investigation periods; therefore, the prosecutor investigates every period as long as there is no agreement, or she does not investigate at any period. Then the question is for what values of the prior belief she does not investigate. If $\theta > \bar{\theta}^N$ she always investigate because $\theta\big((1-q)h+dq\big)+(1-\theta)q(d-c) > d$. If $\theta \leq \bar{\theta}^N$, she does not investigate for $\theta \leq \underline{\theta}^{Public}$.

The prosecutor's expected payoff is

$$u^P = \begin{cases} d & \text{if } \theta \in (0, \underline{\theta}^{Public}] \\ \theta\Big[(1-q)h + qdq\Big] + (1-\theta)dq & \text{if } \theta \in (\underline{\theta}^{Public}, \bar{\theta}^N] \\ \theta\Big[(1-q)h + qd\Big] + (1-\theta)q(d-c) & \text{if } \theta \in (\bar{\theta}^N, 1), \end{cases}$$

and the defendant's expected punishments are

$$u^I = \begin{cases} d & \text{if } \theta \in (0, \underline{\theta}^{Public}] \\ dq & \text{if } \theta \in (\underline{\theta}^{Public}, 1] \end{cases} \quad \text{and} \quad u^G = \begin{cases} d & \text{if } \theta \in (0, \underline{\theta}^{Public}] \\ (1-q)h + qdq & \text{if } \theta \in (\underline{\theta}^{Public}, \bar{\theta}^N] \\ (1-q)h + qd & \text{if } \theta \in (\bar{\theta}^N, 1) \end{cases}$$

The prosecutor's payoff is weakly better off with public investigation for $\theta \leq \underline{\theta}^N$. The payoffs coincide at the limit as $N \to \infty$. The prosecutor is better off with public investigation because it is credible that she is not going to investigate for low values of θ. Therefore the innocent defendant is willing to accept $x = d$.

A.3 Inconclusive Evidence

In this Section I show that the same intuitions in the model are extended to the case in which the evidence is not conclusive regarding the type of defendant's type under a modification of the baseline model. I consider a simplified version of the model, in which the prosecutor always investigates, and there is only one period of negotiation.

Consider further that the probability of finding new evidence is $1 - q^G$ if the defendant is guilty, and $1 - q^I$, if the defendant is innocent, with $q^G, > q^I$. This assumption implies that if the prosecutor does not find new evidence, the posterior belief about the defendant's being guilty is higher than the prior belief. Lastly, consider that the new evidence is $y = h$ with probability π^G if the defendant is guilty, and π^I, if the defendant is innocent, with $\pi^G, > \pi^I$.

If the prosecutor finds evidence $y = h$, she discloses it and offers $x = h$, and both defendant types accept it. If the prosecutor does not find new evidence, there is no disclosure. The defendant's second-order belief, depending on his type, is

$$P^G(\text{d-type} \mid \text{no disclosure}) = 1 - q^G$$
$$P^I(\text{d-type} \mid \text{no disclosure}) = 1 - q^I.$$

The expected punishment for each defendant type at trial is $v^G = dq^G$ and $v^I = dq^I$. In this model, if the prosecutor does not find new evidence, she updates her belief to

$$P(\alpha = G \mid y = d) = \frac{q^G\theta}{q^G\theta + q^I(1-\theta)} \equiv \theta^d.$$

The optimal offer that the d-type prosecutor makes depends on θ^d. If the prosecutor offers the guilty defendant's expected punishment, only the guilty defendant accepts it. Both defendant types accept it if the offer is equal to the innocent defendant's expected punishment. Therefore, the prosecutor offers $x = dq^G$ if $\theta^d > \frac{q^I}{q^G} \equiv \bar{\theta}^{NC}$.

If the prosecutor finds evidence $y = e$, her posterior belief is

$$P(\alpha = G|y = e) = \frac{(1-q^G)\pi^G\theta}{(1-q^G)\pi^G\theta + (1-q^I)\pi^I(1-\theta)} \equiv \theta^e.$$

Consider the case in which $\theta^d > \bar{\theta}^{NC}$ and $\theta^e < \bar{\theta}^{NC}$. The d-type prosecutor makes a high offer to the defendant, but the e-type prefers to make a low offer. The low offer is going to be rejected; the defendant will know that prosecutor has evidence $y = e$ because she is playing a non sequentially rational strategy for the d-type.

The two candidates for optimal strategy for the e-type prosecutor are to disclose $y = e$ and to offer $x = 0$, or make the high offer. The latter case is preferred if $\theta^e dq^G + (1-\theta^e)(-c) \geq 0$ or $\theta^e \geq \frac{c}{dq^G + c}$.

Define:

$$\tilde{\theta}^{NC} \equiv \frac{c}{dq^G + c}.$$

The prosecutor is going to disclose evidence $y = e$ if the following conditions hold:

$$\theta^d > \bar{\theta}^{NC},\ \theta^e < \bar{\theta}^{NC},\ \text{and}\ \theta^e < \tilde{\theta}^{NC}$$

Figure 10 shows the conditions when the prosecutor discloses exculpatory evidence.

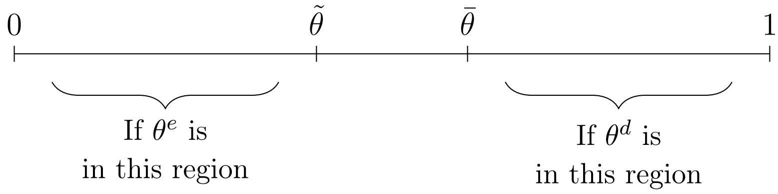

Figure 10: Conditions on posterior belief to disclose exculpatory evidence.

Now, in terms of the prior belief θ, the prosecutor discloses exculpatory evidence if the prior belief is either not too high or too low.[23]

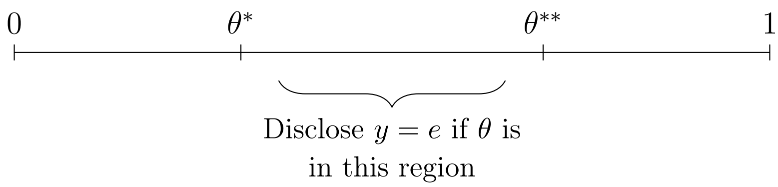

Figure 11: Conditions on prior belief to disclose exculpatory evidence.

Figure 11 shows that, for the inconclusive case, the prior belief cannot be too high to disclose exculpatory evidence, because in that case the prosecutor will still make a high offer if she gets exculpatory evidence.

If the prosecutor does not get new evidence, and $\theta^d > \bar{\theta}^{NC}$, the prosecutor makes an offer that only the guilty defendant accepts. Therefore, if the defendant is innocent, they do not reach an agreement and they go to trial. If the disclosure of evidence is mandatory, they always reach an agreement.

A.4 Bayesian Trial

In this Section, I consider an alternative way to model the trial. Suppose that evidence d does not exist, and the trial assigns a sentence depending on the public posterior belief after exculpatory or incriminating evidence is revealed. The public posterior belief is denoted by θ^T. The defendant gets a sentence h if the evidence is incriminating, 0 if it is exculpatory, and $\theta^T h$ if there is no evidence.

[23] $\theta^* = \frac{q^I \tilde{\theta}}{q^G - \tilde{\theta}(q^G - q^I)}$ and $\theta^{**} = \frac{(1-q^I)\pi^I \tilde{\theta}}{(1-q^G)\pi^G - \tilde{\theta}((1-q^G)\pi^G - (1-q^I)\pi^I)}$

To get a simple intuition, consider the one-period model of Section 3. In this case, the investigation is not a decision, and the prosecutor always investigates. Following the same arguments as in Section 3, the no-evidence-type prosecutor makes an offer θ^T if

$$\theta h \theta^T + (1-\theta)(\theta^T h - c) \geq q\theta^T h$$

In equilibrium, $\theta^T = \theta$ given that not having evidence does not change the prior belief. Therefore, the prosecutor discloses exculpatory evidence to avoid the trial if $\theta \geq \frac{c}{h(1-q)+c}$. This result is qualitatively equivalent to the key insight of the paper.

Nevertheless, considering the prosecutor's decision regarding investigation, the result differs from the model in the main part of the paper.

Lemma 4 *If the investigation is the prosecutor's private information, the prosecutor investigates for any value of the prior belief. If the investigation decision is public, the prosecutor does not investigate for any prior belief.*

Proof. The proof of the Lemma 4 is a comparison of expected payoffs. For the first result, suppose the defendant accepts θd with probability μ. Also, suppose that $\theta h\mu - c(1-\mu) < 0$, that is, if the prosecutor investigates and gets exculpatory evidence, she discloses it. Prosecutor's payoff of no investigation is $V = \theta h\mu(\theta h - c)(1-\mu)$, therefore she deviates to investigate if:

$$V < \theta[(1-q)h + qV] + (1-\theta)qV \quad \Leftrightarrow \quad V < \theta h$$

which is always true. Consider now $\theta h\mu - c(1-\mu) \geq 0$, in this case the prosecutor deviates to investigate if:

$$V < \theta[(1-q)h + qV] + (1-\theta)[(1-q)(\theta h\mu - c(1-\mu)) + qV] \quad \Leftrightarrow \quad \theta < 1 + \frac{c(1-\mu)}{h\mu}$$

which is always true.

For the second result, the prosecutor investigates if the expected payoff of the investigation is higher than θh. The prosecutor's payoffs are $\theta h(1-\theta q(1-q))$ and $\theta h - (1-\theta)qc$ for the cases of no disclosure and disclosure of exculpatory evidence, and both are lower than θh. ■

The intuitive reason for the difference between public and private investigation is that the prosecutor perfectly signals that he has no evidence if he decides not to investigate in the former case. The highest payoff that the prosecutor can get is θh. If the defendant is guilty, he can get at most h with probability θ, and if the defendant is innocent, he does not get more than $q\theta h$ with probability $(1-\theta)$. Therefore if the prosecutor can signal no evidence, she will do it.

However, when the investigation is private information, the incentive to investigate is very high. If the prosecutor gets new evidence, it will be h with probability θ and e with probability $(1-\theta)$. Therefore, at worst, the prosecutor gets $qV + (1-q)\theta h$ if she deviates, which is higher than V because the value of V is increasing in θ. So it is always better to try to get h even if θ is very low. In the model of the main part of the paper, this is not true because V is not a function of θ; therefore, for low values of θ, it is better not to investigate.

A.5 Parametric Assumption

If $c > d(1-q)$ is not satisfied then $\tilde{\theta} \leq 0$. That is, the prosecutor always discloses exculpatory evidence. The intuition is that given a low cost of going to trial with respect to d, the d-type prosecutor always prefers to offer $x = d$ at the last period. That implies the e-type prosecutor never imitates the d-type prosecutor offer; otherwise, she will have a negative payoff to trial.

The condition $h > d\big(q + \frac{c}{c-d(1-q)}\big)$ allows $\underline{\theta}^N < \tilde{\theta}$ for $N \to \infty$ that is the most restrictive case. For the analysis I consider a less restrictive condition (for a general N):

$$h > d\left(\frac{c(q^{\frac{1}{N}} + q) - dq(1-q)}{c - d(1-q)}\right)$$

Note that $q + \frac{c}{c-d(1-q)} > \frac{c(q^{\frac{1}{N}}+q)-dq(1-q)}{c-d(1-q)}$ for any N.
If the above condition is not satisfied, then $\underline{\theta}^N > \tilde{\theta}$. There are two cases.
Case 1: $\underline{\theta}^N > \bar{\theta}^N$. The new cutoff θ such that prosecutor investigates is

$$\underline{\theta}^{*N} = \frac{cq^{\frac{N-1}{N}} + d(1-q^{\frac{N-1}{N}})}{cq^{\frac{N-1}{N}} + d(1-q) + h(1-q^{\frac{N-1}{N}})}$$

Note $\underline{\theta}^{*N}$ is decreasing in N, whit limit:

$$\underline{\theta}^{*} = \frac{cq + d(1-q)}{cq + d(1-q) + h(1-q)}$$

Here the prosecutor either always discloses $y = e$ if $\theta > \underline{\theta}^N$, or reaches an immediate agreement with the defendant if $\theta \leq \underline{\theta}^N$.

Case 2: $\underline{\theta}^N \in (\tilde{\theta}, \bar{\theta}^N]$. In this case the results described in the main part of the paper hold.

B Proofs

Proposition 1. The prosecutor's payoff if she discloses $x = h$ is h. If the prosecutor does not disclose $y = h$, the guilty defendant's belief about the prosecutor's type is P_G(d$-$*type* | *no-disclosure*) = 1, it implies the guilty defendant does not accept anything higher than $x = d$ that gives the prosecutor a payoff of at most d.

If $\theta \leq \tilde{\theta}$, the prosecutor offers $x = dq$ if $y = d$ after the investigation. If prosecutor deviates to offer $x > d$, the offer is rejected and she gets a payoff of $d - c < dq$. If she deviates to $x \in (dq, d]$ the offer is only accepted by the guilty defendant and rejected by the innocent. Therefore her expected payoff is $\theta x + (1 - \theta)(d - c)$ which is lower than dq because $\theta x + (1-\theta)(d-c) \leq \theta d + (1-\theta)(d-c) = d - c(1-\theta) < dq$ for $\theta < \tilde{\theta}$. If prosecutor deviates to offer $x < dq$, both defendant types accept the offer and the prosecutor gets a payoff lower than dq. Therefore there is no profitable deviation.

If $\theta \leq \tilde{\theta}$ and $y = e$, the best response for the e-type prosecutor is to mimic the d-type prosecutor. Note that the e-type prosecutor knows the defendant is innocent. Therefore, if she offers $x > dq$, the defendant will reject the offer, and she gets a payoff of $-c$. If she offers $x < dq$, the offer is rejected, and she gets $-c$. The offer is rejected because the innocent defendant that gets an offer $x < dq$ updates his belief about the prosecutor's type to the persecutor being a e-type prosecutor with probability 1.

For $\theta \leq \tilde{\theta}$, the guilty defendant always accepts dq because her expected punishment of going to the trial is at least d. The innocent defendant accepts $x = dq$ because her second-order belief about the prosecutor's evidence is e with probability $1 - q$ and d with probability q. Therefore his expected punishment at trial is dq. The innocent defendant rejects $x < dq$ because the d-type prosecutor never sends that offer. Accordingly, he updates his belief to the prosecutor being e-type, and therefore, her expected punishment at trial is 0.

Consider $\theta > \tilde{\theta}$. The prosecutor offers $x = d$ if $y = d$. If she deviates to offer $x > d$, the offer is rejected for sure and she gets a payoff of $d - c < \theta d + (1 - \theta)(d - c)$. If she deviates to $x < d$, only the guilty defendant accepts it and the prosecutor gets an expected payoff of $\theta x + (1 - \theta)(d - c) < \theta d + (1 - \theta)(d - c)$.

The prosecutor discloses e and offers $x = 0$ if $y = e$. If the prosecutor offers $x > 0$, the offer is rejected, and the prosecutor gets $-c$. If the prosecutor does not discloses $y = e$, and she offers $x \geq d$, the offer is rejected for sure, and she gets $-c$ at trial. Suppose she offers $x < d$, the innocent defendant updates his belief about the prosecutor type to be e-type and therefore rejects the offer, and the prosecutor gets $-c$. If the prosecutor does not disclose $y = e$ and offers $x = 0$, the payoff is the same as disclosing it and offering $x = 0$.

For $\theta > \tilde{\theta}$ the guilty defendant always accepts d because his expected punishment of going to the trial is at least d. The innocent defendant does not accept $x > 0$. If there is no disclosure, a deviation to accept $x = d$ generates the same expected punishment at the trial, so it is not a profitable deviation. Note that accepting $x = d$ cannot be an equilibrium, because in that case, the prosecutor with $y = e$ deviates to no-disclosing. Any offer $x \in (0, d)$ reveals the prosecutor has $y = e$.

Lemma 1. The prosecutor's strategy is as follows. On the equilibrium path, she only investigates at $n = 1$. If she gets $y = h$, she discloses it and offers $x = h$. If she gets $y = e$ or does not get any new evidence, she does not disclose it and offers $x = dq^{\frac{1}{N}}$ for all n.

The guilty defendant's strategy, if there is disclosure of $y = h$, is $\mu^G(x) = 1$ if $x \leq h$, and $\mu^G(x) = 0$ otherwise. If there is no-disclosure of $y = h$; $\mu^G(x) = 1$ if $x \leq d$, and $\mu^G(x) = 0$ otherwise for all n. The innocent defendant's strategy, if there is disclosure of $y = e$, is $\mu^I(x) = 1$ if $x \leq 0$, and $\mu^I(x) = 0$ otherwise. If there is no disclosure of $y = e$; $\mu^I(x) = 1$ if $x \leq dq^{\frac{1}{N}}$ and $\mu^I(x) = 0$ otherwise for all n.

Disclosing $x = h$ is the best response. The prosecutor's continuation value if she discloses $x = h$ is $v^P = h$. If the prosecutor does not disclose $y = h$ the guilty defendant's belief about the prosecutor's type is $P_G(\text{d}-\textit{type} \mid \textit{no-disclosure}\ ) = 1$, it implies the guilty defendant does not accept anything higher than $x = d$, that gives the prosecutor a continuation value $v^P = dq^{\frac{1}{N}}$.

The prosecutor expected continuation payoff at the end of $n = 1$, if $y \in \{e, d\}$ is $dq^{\frac{1}{N}}$. The innocent defendant and the guilty defendant expected punishment are: $v^I = dq^{\frac{1}{N}}$ and $v^G = dq^{\frac{1}{N}}$.

The guilty defendant does not deviate to rejection because if the prosecutor observes a rejection, she will update her belief to $\theta' = 0$; she does not investigate the following periods and offers $x = dq^{\frac{1}{N}}$ every subsequent period. Therefore, the guilty defendant is not better off. The same applies to the innocent defendant; if there is a rejection, the prosecutor is not going to investigate, and she will offer $x = dq^{\frac{1}{N}}$ next period.

Note that it is not possible to have a different belief than $\theta' = 0$ when there is a rejection, because if $\theta' > 0$ the prosecutor will investigates at least one more period, that bring an expected payoff of at least $(1 - q^{\frac{1}{N}})h + q^{\frac{1}{N}}dq^{\frac{2}{N}}$ to her. It is larger than $dq^{\frac{1}{N}}$. Therefore the guilty defendant will reject with probability one. Thus, it is not possible to have $\theta' > 0$.

The prosecutor does not deviate to offer less than $x = dq^{\frac{1}{N}}$ because it brings her a lower payoff. If $x > dq^{\frac{1}{N}}$ the innocent defendant will reject it because $dq^{\frac{1}{N}}$ is his highest expected punishment when there is no further investigation, so he will never accept a larger offer. It cannot be that the guilty defendant accepts it with probability one because, in that case, $\theta' = 0$ and the prosecutor will decrease the offer in later periods. Therefore the guilty defendant is better of rejecting it. If the guilty defendant accepts it with probability $\mu^G < 1$ such that there is investigation in future periods, the prosecutor is worse off because if there is at least one more investigation her payoff will be at most $v^P = \theta\Big((1-q^{\frac{1}{N}})h + q^{\frac{1}{N}} dq^{\frac{2}{N}}\Big) + (1-\theta)dq^{\frac{2}{N}}$ that is lower than $dq^{\frac{1}{N}}$ when $\theta \leq \tilde{\theta}^N$. Therefore, there is no profitable deviation.

The prosecutor investigates the first period is an equilibrium, otherwise she gets $dq^{\frac{1}{N}}$ instead of $\theta\Big((1-q^{\frac{1}{N}})h + q^{\frac{1}{N}} dq^{\frac{1}{N}}\Big)(1-\theta)dq^{\frac{1}{N}}$ that is larger than $dq^{\frac{1}{N}}$ when $\theta \leq \bar{\theta}^N$. Note that off the equilibrium path $\theta' = 0$ for $n > 1$, that implies the prosecutor never deviates to investigate the following periods.

Proposition 2. The players strategies depend on the value of θ. If $\theta \in \big(\underline{\theta}^N, \tilde{\theta}\big]$ the prosecutor's strategy is to investigates every period. If $y = h$, she discloses it and offers $x = h$. If $y \in \{e, d\}$, she does not disclose it and offers $x = h$ if $n \in \{1, ..., N-1\}$, and $x = dq$ if $n = N$.

The guilty defendant's strategy, if there is disclosure of $y = h$, is $\mu^G(x) = 1$ if $x \leq h$, and $\mu^G(x) = 0$ otherwise. If there is no disclosure of $y = h$; $\mu^G(x) = 1$ if $x < (1 - q^{\frac{N-n}{N}})h + q^{\frac{N-n}{N}}dq$, and $\mu^G(x) = 0$ otherwise for $n \in \{1, ..., N-1\}$, and $\mu^G(x) = 1$ if $x \leq d$, and $\mu^G(x) = 0$ otherwise if $n = N$. The innocent defendant's strategy, if there if disclosure of $y = e$, is $\mu^I(x) = 1$ if $x \leq 0$, and $\mu^I(x) = 0$ otherwise. If there is no disclosure of $y = e$; $\mu^I(x) = 1$ if $x \leq dq^{\frac{n}{N}}$ and $\mu^I(x) = 0$ otherwise for all n.

If $\theta \in \big(\tilde{\theta}, \bar{\theta}^N\big]$, the prosecutor's equilibrium strategy is to investigates every period. If $y = h$, she discloses it and offers $x = h$. If $y \in \{e, d\}$, she does not disclose it and offers $x = (1 - q^{\frac{N-1}{N}})h + q^{\frac{N-1}{N}}dq$ if $n = 1$, $x = h$ if $n \in \{1, ..., N-1\}$, and $x = dq$ if $n = N$.

The guilty defendant's strategy, if disclosure of $y = h$, is $\mu^G(x) = 1$ if $x \leq h$, and $\mu^G(x) = 0$ otherwise. If there is no disclosure of $y = h$; $\mu^G(x) = 1$ if $x < (1 - q^{\frac{N-n}{N}})h + q^{\frac{N-n}{N}}dq$, $\mu^G(x) = \frac{\theta - \tilde{\theta}}{\theta(1-\tilde{\theta})}$ if $x = (1 - q^{\frac{N-n}{N}})h + q^{\frac{N-n}{N}}dq$ and $\mu^G(x) = 0$ otherwise for $n \in \{1, ..., N-1\}$, and $\mu^G(x) = 1$ if $x \leq d$, and $\mu^G(x) = 0$ otherwise if $n = N$. The innocent defendant's strategy, if there is disclosure of $y = e$, is $\mu^I(x) = 1$ if $x \leq 0$, and $\mu^I(x) = 0$ otherwise. If there if no disclosure of $y = e$; $\mu^I(x) = 1$ if $x \leq dq^{\frac{n}{N}}$ and $\mu^I(x) = 0$ otherwise for all n.

Finally, if $\theta \in \big(\bar{\theta}^N, 1\big)$, the prosecutor's investigates every period. If $y = h$ or $y = e$, she discloses it and offers $x = h$ or $x = 0$ respectively. If $y = d$, she offers $x = h$ if $n \in \{1, ..., N-1\}$, and $x = d$ if $n = N$. The guilty defendant's strategy, if there is disclosure of $y = h$, is $\mu^G(x) = 1$ if $x < h$, and $\mu^G(x) = 0$ otherwise. If there is no disclosure of $y = h$; $\mu^G(x) = 1$ if $x < (1 - q^{\frac{N-n}{N}})h + q^{\frac{N-n}{N}}dq$, and $\mu^G(x) = 0$ otherwise for $n \in \{1, ..., N-1\}$, and $\mu^G(x) = 1$ if $x \leq d$, and $\mu^G(x) = 0$ otherwise if $n = N$. The innocent defendant's strategy is $\mu^I(x) = 1$ if $x \leq 0$, and $\mu^I(x) = 0$ otherwise for all n.

Remark: If the prosecutor hides exculpatory evidence, the innocent defendant rejects $x = d$ with probability one. Also, his belief about the prosecutor type is $P^I(d\text{−type} \mid x < d) = 0$ and $P^I(d - \text{type} \mid x = d) = 1$. This implies if the prosecutor deviates and offers $x \in (0, d)$, the innocent defendant rejects the offer because she is better off going to trial.

Note that for every period n it is not possible to have $y = h$ at the beginning of the period on the equilibrium path. This is because the prosecutor discloses it and offer $x = y$ at the end of the period that she gets it. Also, for $n \leq 2$ it is not possible that the prosecutor's belief about the defendant's type belongs to the interval $(0, \underline{\theta}^N]$ or $(\bar{\theta}, \tilde{\theta}^N]$. This is because on the equilibrium path, in the first period the prosecutor either ends the game if $(0, \underline{\theta}^N]$ or updates her belief to $\theta' = \tilde{\theta}$ if $\left(\tilde{\theta}, \bar{\theta}^N\right]$.

For this Section consider the following notation: y_f^n represents the evidence that the prosecutor has after the investigation at period n.

I. Last period before trial ($n = N$): Following on-path strategies the prosecutor can only have $y^N \in \{e, d\}$ at the beginning of the period. If $y_f^N = h$; to disclose the evidence and offer $x^N = h$, and $\mu^G(h) = 1$ is an equilibrium. The guilty defendant's continuation punishment if he rejects $x = h$ is $v^G = h$, so he is indifferent.

Disclosing $x = h$ is the best response. The prosecutor's continuation value if she discloses $x = h$ is $v^P = h$. If the prosecutor does not disclose $y = h$, the guilty defendant's belief about the prosecutor's type is $P_G(\text{d}-\textit{type} \mid \textit{no-disclosure}\ ) = 1$, it implies the guilty defendant does not accept anything higher than $x = d$ that gives the prosecutor a continuation value $v^P = d$.

Case $\theta \in (\underline{\theta}^N, \tilde{\theta}]$. If $y_f^N = d$, the continuation punishments are $v^G = d$ and $v^I = dq$. The prosecutor's optimal offer is either $x = v^I$ such that both defendant types accept it $\mu^I(dq) = \mu^G(dq) = 1$, or $x = v^G$ such that only the guilty defendant accepts it $\mu^I(d) = 0, \mu^G(d) = 1$. The prosecutor is better off offering the innocent defendant's continuation punishment because it brings her an expected payoff of qd, that is larger than θd when $\theta \leq \tilde{\theta}$.

If the outcome of the investigation is $y_f^N = e$, disclosing it gives the prosecutor a continuation payoff of $v^P = 0$, because the innocent defendant's continuation punishment is zero. If she does not disclose it, she can offer $x = dq$ that the innocent defendant accepts.

Case $\theta \in (\bar{\theta}^N, 1)$. Suppose $y_f^N = d$. Continuation punishments are $v^G = d$ and $v^I = dq$. The prosecutor's optimal offer is either $x = v^I$ such that both defendant types accept it with $\mu^I(dq) = \mu^G(dq) = 1$, or $x = v^G$ such that only guilty defendant accepts it $\mu^I(d) = 0, \mu^G(d) = 1$. The prosecutor is better off offering the guilty defendant's continuation punishment because it brings her an expected payoff of θd, instead of dq when $\theta > \bar{\theta}^N$. Note this implies the prosecutor offers $x = d$ and guilty defendant accepts it.

If $y_f^N = e$, disclosing it is an equilibrium. It cannot be an equilibrium where a e-type prosecutor can successfully hide evidence and get a payoff higher than zero. If the prosecutor has evidence $y = d$, she will offer $x = d$ because any other offer is strictly dominated. Therefore, if there is no disclosure and the innocent defendant gets an offer $x \in (0, d)$, he will update his belief about the prosecutor type to $P_I(y = d \mid x \in (0, d)) = 0$, because otherwise she would have offered $x = d$, and then the innocent defendant will reject the offer.

Finally, if $y^N = d$ at the beginning of period N, the prosecutor does not deviates from investigate, otherwise $v^P = dq$ instead if $v^P = \theta[(1-q^{\frac{1}{N}})h + q^{\frac{1}{N}}dq] + (1-\theta)dq$ if $\theta \leq \tilde{\theta}$, and $v^P = \theta d + (1-\theta)(d-c)$ instead of $v^P = \theta[(1-q^{\frac{1}{N}})h + q^{\frac{1}{N}}d] + (1-\theta)q^{\frac{1}{N}}(d-c)$ if $\theta > \bar{\theta}^N$. Note that $\theta d + (1-\theta)(d-c)$ is larger than $\theta[(1-q^{\frac{1}{N}})h + q^{\frac{1}{N}}d] + (1-\theta)q^{\frac{1}{N}}d$ for $\theta > \frac{d-c}{h-c}$, and note further that $\frac{d-c}{h-c}$ is always lower than $\tilde{\theta}$ given the parametric assumptions, therefore $\frac{d-c}{h-c} < \bar{\theta}^N$.

II. Intermediate periods ($1 < n < N$): Following on-path strategies the prosecutor can only have $y^n \in \{e, d\}$. Suppose the prosecutor investigates and gets evidence $y = h$; to disclose the evidence and offer $x^n = h$, and $\mu^G(x = h) = 1$ is an equilibrium. The guilty defendant's continuation punishment if he rejects $x = h$ is $v^G = h$, so he is indifferent.

To disclose $x = h$ is best response. The prosecutor's continuation value if she discloses $x = h$ is $v^P = h$. If the prosecutor does not disclose $x = h$, $P_G(y = d \mid \textit{no-disclosure}\ ) = 1$, it implies the guilty-defendant does not accept anything higher than $x = (1-q^{\frac{N-n}{N}})h + q^{\frac{N-n}{N}}dq$.

Case $\theta \in (\underline{\theta}^N, \tilde{\theta}]$. Suppose $y^n_f = d$. The continuation punishments are $v^G = (1-q^{\frac{N-n}{N}})h + q^{\frac{N-n}{N}}dq$ and $v^I = dq$. The prosecutor's continuation payoff is $v^P = \theta\big((1-q^{\frac{N-n}{N}})h + q^{\frac{N-n}{N}}dq\big) + (1-\theta)dq$.

Any offer x that is not rejected by both defendant types is not a profitable deviation for the prosecutor. Consider an offer that both types accept. The highest offer—the one that maximizes the prosecutor's payoff—that both defendant types accept, is $qd^{\frac{n}{N}}$. This is not a profitable deviation from offering x such that both defendant types reject, because $\theta\big((1-q^{\frac{N-n}{N}})h + q^{\frac{N-n}{N}}dq\big) + (1-\theta)dq$ is higher than $qd^{\frac{n}{N}}$ when $\theta > \underline{\theta}^N$. The innocent defendant rejects any higher offer because $dq^{\frac{n}{N}}$ is the highest continuation punishment that the innocent defendant can get, that it is reached when the prosecutor does not investigates any further period.

The highest offer that only the guilty defendant might accept is his continuation punishment minus $\epsilon \to 0$, $x' = (1-q^{\frac{N-n}{N}})h + q^{\frac{N-n}{N}}dq - \epsilon$. This offer is not a profitable deviation because the prosecutor's payoff under the deviation is at most $\theta\big((1-q^{\frac{N-n}{N}})h + q^{\frac{N-n}{N}}dq\big) + (1-\theta)dq$.

It is not an equilibrium that the prosecutor offers x', the guilty defendant accepts with probability $\mu^G \in (0,1]$, and the innocent defendant rejects it, such that $\theta' < \underline{\theta}^N$ if a rejection is observed. In that case the guilty defendant anticipates the updating and deviates to reject x, because the continuation punishment if $\theta' < \underline{\theta}^N$ is lower. Any other strategy by the guilty defendant provides the prosecutor a continuation payoff bounded by $\theta\big((1-q^{\frac{N-n}{N}})h + q^{\frac{N-n}{N}}dq\big) + (1-\theta)dq$. If $\mu^G \in [0,1)$, such that $\theta' \in (\underline{\theta}^N, \theta]$ if prosecutor observes a rejection, the prosecutor gets a expected payoff of $\theta\big((1-q^{\frac{N-n}{N}})h + q^{\frac{N-n}{N}}dq - \mu^G\epsilon\big) + (1-\theta)dq$ under the deviation offer.

If $y^n_f = e$ the prosecutor equilibrium strategy is to mimic a d-type prosecutor. The relevant deviation to check this is an equilibrium is not to mimic the d-type prosecutor. If the prosecutor discloses $x = e$ or offers $x < v^G$, the innocent defendant updates $P_I(y = d \mid x < v^G) = 0$, that gives a lower expected continuation payoff for the prosecutor. Therefore, to mimic a d-type prosecutor is an equilibrium.

The prosecutor investigates at the beginning of n is an equilibrium, because it gives him an expected payoff of $v^P = \theta[(1 - q^{\frac{N+1-n}{N}})h + q^{\frac{N+1-n}{N}} dq] + (1-\theta)dq$ that is higher than the one shot no-investigation payoff $v^P = \theta[(1 - q^{\frac{N-n}{N}})h + q^{\frac{N-n}{N}} dq] + (1-\theta)dq$.

Case $\theta \in (\bar{\theta}^N, 1)$: Consider $\theta \in (\tilde{\theta}^N, 1)$. Suppose $y_f^n = d$. The continuation punishments are $v^G = (1 - q^{\frac{N-n}{N}})h + q^{\frac{N-n}{N}} d$ and $v^I = dq$. The prosecutor's continuation value is $v^P = \theta\Big((1 - q^{\frac{N-n}{N}})h + q^{\frac{N-n}{N}}))d\Big)$.

The equilibrium strategy for the prosecutor is to offer $x > (1 - q^{\frac{N-n}{N}})h + q^{\frac{N-n}{N}} d$ such that is rejected by both defendant types. If she deviates to offer x such that both types accepts, she has to offer $qd^{\frac{n}{N}}$ as analyzed above. This is not a profitable deviation because $qd^{\frac{n}{N}}$ is lower than $\theta\Big((1 - q^{\frac{N-n}{N}})h + q^{\frac{N-n}{N}}))d\Big) + (1-\theta)q(d-c)$ when $\theta > \bar{\theta}^N$.

As before, the highest offer that only the guilty defendant accepts is his continuation punishment minus $\epsilon \to 0$, $x' = (1 - q^{\frac{N-n}{N}})h + q^{\frac{N-n}{N}} d - \epsilon$. The induced continuation equilibrium gives an expected payoff to the prosecutor of at most the same payoff than following the equilibrium strategy.

It is not an equilibrium that the prosecutor offers x', the guilty defendant accepts it with probability $\mu^G \in (0, 1]$, and the innocent defendant rejects such that $\theta' < \bar{\theta}^N$ if a rejection is observed. In that case the guilty defendant anticipates the updating and deviates to reject x' because his continuation punishment is lower if $\theta' < \bar{\theta}^N$. This deviation gives the prosecutor an expected payoff of $dq^{\frac{n}{N}}$ if $\theta' \leq \underline{\theta}^N$ and $\theta\big((1 - q^{\frac{N-n}{N}})h + q^{\frac{N-n}{N}} dq\big) + (1-\theta)dq$ if $\theta' \in (\underline{\theta}^N, \bar{\theta}^N]$.

Any other strategy by the guilty defendant provides the prosecutor a continuation payoff bounded by $\theta\Big((1 - q^{\frac{N-n}{N}})h + q^{\frac{N-n}{N}}))d\Big) + (1-\theta)q(d-c)$. If $\mu^G \in [0, 1)$, such that $\theta' \in (\bar{\theta}^N, \theta]$ if prosecutor observes a rejection, the prosecutor gets a expected payoff of $\theta\big((1 - q^{\frac{N-n}{N}})h + q^{\frac{N-n}{N}} d - \mu^G \epsilon\big) + (1-\theta)q(d-c)$ under the deviation offer.

If $y_f^n = e$, the equilibrium strategy for the prosecutor is to disclose the evidence and offer $x = 0$. If the prosecutor deviates to no-disclosure and to offer $x = 0$, it brings the same payoffs than to disclose the evidence and offer $x = 0$, therefore is not a profitable deviation. If the prosecutor does not disclose and offer $x \in (0, d]$ the innocent defendant will reject it, because he will updated his belief to $P_I(y = d \mid \text{no-disclosure and } x \in (0, d]) = 0$ given that a d-type prosecutor never offers less than d.

The prosecutor investigates at the beginning of n is an equilibrium, because it gives him an expected payoff of $v^P = \theta[(1 - q^{\frac{N+1-n}{N}})h + q^{\frac{N+1-n}{N}} d] + (1-\theta)q^{\frac{N+1-n}{N}}(d-c)$ that is higher than the one shot no-investigation payoff $v^P = \theta[(1 - q^{\frac{N-n}{N}})h + q^{\frac{N-n}{N}} d] + (1-\theta)q^{\frac{N-n}{N}}(d-c)$. Note that $\theta[(1 - q^{\frac{N+1-n}{N}})h + q^{\frac{N+1-n}{N}} d] + (1-\theta)q^{\frac{N+1-n}{N}}(d-c)$ is larger than $\theta[(1 - q^{\frac{N-n}{N}})h + q^{\frac{N-n}{N}} d] + (1-\theta)q^{\frac{N-n}{N}}(d-c)$ for $\theta > \frac{d-c}{h-c}$, and note further that $\frac{d-c}{h-c}$ is always lower than $\tilde{\theta}$ given the parametric assumptions, therefore $\frac{d-c}{h-c} < \bar{\theta}^N$.

III. First period ($n = 1$): Suppose the prosecutor investigates and gets evidence $y = h$. To disclose the evidence and offer $x = h$, and $\mu^G(h) = 1$ is the equilibrium. There is no profitable deviation: the guilty defendant's continuation punishment if he rejects $x = h$ is $v^G = h$, so he is indifferent.

To disclose $x = h$ is best response. The prosecutor continuation value if she discloses $x = h$ is $v^P = h$. If the prosecutor does not disclose $x = h$, $P_G(y = d \mid \textit{no-disclosure}\ ) = 1$, it implies the guilty defendant does not accept anything higher than $x = d$. In that case the prosecutor makes an offer that is rejected for sure. At $n + 1$ the prosecutor discloses $y = h$. If the prosecutor never discloses, her continuation value is $v^P = d$, that is lower than h. As before, the prosecutor is indifferent between disclosing $x = h$ at n or at $n+1$; I assume the prosecutor discloses it as soon as she gets it.

The cases $\theta \in (\underline{\theta}^N, \tilde{\theta}]$ and $\theta \in (\bar{\theta}^N, 1)$ are the same that when $n \in (1, N)$ analyze above.

Case $\theta \in (\tilde{\theta}, \bar{\theta}^N]$: The equilibrium is to investigate the first period and to offer $x = (1 - q^{\frac{N-1}{N}})h + q^{\frac{N-1}{N}} dq$ if $y \in \{e, d\}$. The guilty defendant accepts it with probability $\mu^G(x) = \frac{\theta - \bar{\theta}}{\theta(1-\bar{\theta})}$. The innocent defendant rejects it. The prosecutor updates her belief to $\theta' = \bar{\theta}$.

The guilty defendant does not deviate because he gets the same expected payoff as accepting it if he rejects the offer. If the innocent defendant accepts the offer, he is worse off.

The prosecutor does not deviate; if $x < (1 - q^{\frac{N-1}{N}})h + q^{\frac{N-1}{N}} dq$ the guilty defendant accepts the offer for sure, but payoff is lower. Also, it cannot be an equilibrium, because if guilty defendant accepts it for sure, then the prosecutor does not investigates anymore because $\theta' = 0$; therefore the guilty defendant deviates to rejection. If $x \in [(1 - q^{\frac{N-1}{N}})h + q^{\frac{N-1}{N}} dq, (1 - q^{\frac{N-1}{N}})h + q^{\frac{N-1}{N}} d)$ the guilty defendant rejects it given that the continuation punishment when $\theta = \bar{\theta}$ is lower. If $\theta' > \bar{\theta}$ then the prosecutor accepts it; however it is not profitable for the prosecutor when $\theta \leq \bar{\theta}^N$. Also, it cannot be an equilibrium because if the guilty defendant accepts for sure, then the prosecutor does not investigates anymore because $\theta' = 0$; therefore the guilty defendant deviates to rejection. If $x \geq (1 - q^{\frac{N-1}{N}})h + q^{\frac{N-1}{N}} d)$ and $\theta = \tilde{\theta}$ if there is a rejection; the guilty defendant always rejects the offer and the prosecutor is worse off.

If the prosecutor delays the offer x such that $\theta = \tilde{\theta}$, for some values of θ she will be indifferent but for others she will be worse. Suppose the prosecutor delays the offer x to period $n > 1$, at n there were n investigations, so the offer that makes the guilty defendant indifferent is $x = (1 - q^{\frac{N-n}{N}})h + q^{\frac{N-n}{N}} dq$. The prosecutor is willing to make this offer if:

$$\theta\Big((1-q^{\frac{N-n}{N}})h+q^{\frac{N-n}{N}}dq\Big)+(1-\theta)dq\geq\Big((1-q^{\frac{N-n}{N}})h+q^{\frac{N-n}{N}}d\Big)$$
$$\iff\quad\theta\leq\frac{q^{\frac{N-n}{N}}}{q^{\frac{N-n}{N}}+q(1-q^{\frac{N-n}{N}})}\equiv\tilde{\theta}'$$

Note that $\tilde{\theta}'<\bar{\theta}^N$ for $n>1$. This implies that if the prosecutor waits until period n, she is not going to separate the guilty defendant from the innocent if $\theta\in(\tilde{\theta}',\bar{\theta}^N]$. For values $\theta\in(\tilde{\theta},\tilde{\theta}']$ the prosecutor gets the same payoff making the offer x at the first period or waiting until n. For values $\theta\in(\tilde{\theta}',\bar{\theta}^N]$ the prosecutor is worse off waiting until n, because her payoff of making the offer at the first period is: $\theta\Big((1-q^{\frac{N-1}{N}})h+q^{\frac{N-1}{N}}dq\Big)+(1-\theta)dq$ that is larger than waiting until n, where the payoff is $\Big((1-q^{\frac{N-}{N}})h+q^{\frac{N-1}{N}}d\Big)$.

In conclusion, the prosecutor does not deviates and offers $x=(1-q^{\frac{N-1}{N}})h+q^{\frac{N-1}{N}}dq$ at period 1.

Investigation the first period is an equilibrium, because otherwise her payoff is: $\theta\Big((1-q^{\frac{N-1}{N}})h+q^{\frac{N-1}{N}}dq\Big)+(1-\theta)dq$ instead of $\theta\Big((1-q)h+qdq\Big)+(1-\theta)dq$ from the ex ante perspective at the beginning of period 1.

Proposition 3. For $\theta\in(\underline{\theta},\bar{\theta}]$, the prosecutor makes an offer that is accepted by both defendant types at $t=T$, therefore the game ends for sure at T. Note that at $t<T$ the game ends only if the prosecutor gets $y=h$. That happens with probability $1-e^{-\lambda\frac{T-\epsilon}{T}}$ if $\theta\in(\underline{\theta},\tilde{\theta}]$, and $1-e^{-\lambda\frac{T-\epsilon}{T}}(1-\mu^G)$ if $\theta\in(\tilde{\theta},\bar{\theta}]$.

For $\theta\in(\bar{\theta},1]$, at $t=T$ the d-type prosecutor makes an offer that is rejected by the innocent defendant, therefore there is no mass point. If the defendant is guilty, he accepts the offer that the d-type prosecutor makes at $t=T$. Note that at $T-\epsilon$ the game ends if the defendant is guilty only if the prosecutor gets $y=h$, it happens with probability $1-e^{-\lambda\frac{T-\epsilon}{T}}$ if $\theta\in(\bar{\theta},1]$.

Proposition 4. If $\theta\leq\frac{d}{h}$: At period n the prosecutor expected payoff is: $v^P=d$. If the prosecutor one-shot deviates at n, her payoff is:

$$\theta((1-q^{\frac{1}{N}})h+q^{\frac{1}{N}}d)+(1-\theta)q^{\frac{1}{N}}d$$

This is larger than d if: $\theta>\frac{d}{h}$, but $\theta\leq\frac{d}{h}$ by assumption. Hence, it is not a profitable deviation.

If $\theta>\frac{d}{h}$: At period n the prosecutor expected payoff is:

$$\theta((1-q^{\frac{N-n+1}{N}})h+q^{\frac{N-n+1}{N}}d)+(1-\theta)q^{\frac{N-n+1}{N}}d$$

If the prosecutor one-shot deviates at n, her payoff will be:

$$\theta((1-q^{\frac{N-n}{N}})h + q^{\frac{N-n}{N}}d) + (1-\theta)q^{\frac{N-n}{N}}d$$

The deviation payoff is larger than no-deviation if $\theta < \frac{d}{h}$. However $\theta > \frac{d}{h}$ by assumption. Therefore, there is no profitable deviation.

Proposition 5. The innocent defendant expected punishment for the innocent defendant in the frequent-offer-limit voluntary disclosure case is

$$u^I = \begin{cases} d & \text{if } \theta \in (0, \underline{\theta}] \\ dq & \text{if } \theta \in (\underline{\theta}, 1] \end{cases}$$

and in the mandatory disclosure case is:

$$u^I = \begin{cases} d & \text{if } \theta \in (0, \underline{\theta}^{BR}] \\ dq & \text{if } \theta \in (\underline{\theta}^{BR}, 1) \end{cases}$$

For $\theta < \underline{\theta}^{BR}$ the expected payoff are the same. For $\theta \in (\underline{\theta}^{BR}, \underline{\theta}]$ under mandatory disclosure the expected punishment is dq that is smaller than d (the expected punishment under voluntary disclosure). For $\theta \in (\underline{\theta}^{BR}, 1)$ the expected punishments are the same.

The guilty defendant expected punishment for the innocent defendant in the voluntary disclosure case is:

$$u^G = \begin{cases} d & \text{if } \theta \in (0, \underline{\theta}] \\ (1-q)h + qdq & \text{if } \theta \in (\underline{\theta}, \bar{\theta}] \\ (1-q)h + qd & \text{if } \theta \in (\bar{\theta}, 1) \end{cases}$$

and in the mandatory case is:

$$u^G = \begin{cases} d & \text{if } \theta \in (0, \underline{\theta}^{BR}] \\ (1-q)h + qd & \text{if } \theta \in (\underline{\theta}^{BR}, 1) \end{cases}$$

For $\theta < \underline{\theta}^{BR}$ the expected punishments are the same. For $\theta \in (\underline{\theta}^{BR}, \underline{\theta}]$ under mandatory disclosure the expected punishment is $(1-q)h + qd$ that is larger than the voluntary disclosure expected payoff d.

For $\theta \in (\underline{\theta}, \tilde{\theta}]$, the guilty defendant expected punishment under mandatory disclosure is $(1-q)h+qd$ that is larger than expected payoff with voluntary disclosure $(1-q)h+qdq$. For $\theta \in (\tilde{\theta}, 1)$, the expected punishment are the same.

Proposition 6. The prosecutor's payoffs in the frequent-offer-limit case $N \to \infty$ with voluntary disclosure of evidence is:

$$u^P = \begin{cases} d & \text{if } \theta \in (0, \underline{\theta}] \\ \theta\Big[(1-q)h + qdq\Big] + (1-\theta)dq & \text{if } \theta \in (\underline{\theta}, \bar{\theta}] \\ \theta\Big[(1-q)h + qd\Big] & \text{if } \theta \in (\bar{\theta}, 1) \end{cases}$$

while in the mandatory disclosure of evidence is:

$$u^P = \begin{cases} d & \text{if } \theta \in (0, \underline{\theta}^{BR}] \\ \theta\Big((1-q)h + dq\Big) + (1-\theta)dq & \text{if } \theta \in (\underline{\theta}^{BR}, 1) \end{cases}$$

Note $\underline{\theta}^{BR} < \underline{\theta}$ given that $\frac{d}{h} < \frac{d}{h-dq}$. For $\theta \leq \underline{\theta}^{BR}$ the prosecutor payoff are the same. For $\theta \in (\underline{\theta}^{BR}, \underline{\theta}]$ the prosecutor payoff for no-investigation is d that is larger than the prosecutor payoff of always-investigation $\theta\Big((1-q)h + dq\Big) + (1-\theta)dq$ when $\theta > \underline{\theta}^{BR}$.

For $\theta \in (\underline{\theta}, \tilde{\theta}]$, the prosecutor payoff with mandatory disclosure is: $\theta\Big((1-q)h + dq\Big) + (1-\theta)dq$ that is larger than $\theta\Big[(1-q)h + qdq\Big] + (1-\theta)dq$. For $\theta \in (\tilde{\theta}, 1)$, the mandatory disclosure payoff for the prosecutor is $\theta\Big((1-q)h + dq\Big) + (1-\theta)dq$ that is larger than $\theta\Big((1-q)h + dq\Big) + (1-\theta)dq$ when $\theta < 1$.

Proposition 7. Call $\mu_n^I(x)$ the probability that the innocent defendant accepts the offer x at period n. The prosecutor expected payoff of the equilibrium strategies is $\theta d + (1-\theta)d\mu_{1,..,N}^I$, where $\mu_{1,..,N}^I$ is the probability of accepting $x = d$ at any period between 1 and N.
For $\mu_N^I \in [0, \frac{c}{d+c})$: If the prosecutor one-shot deviates at period $n = 1$, her payoff is

$$\theta\Big((1 - q^{\frac{1}{N}})h + q^{\frac{1}{N}}d\Big) + (1-\theta)\Big((1 - q^{\frac{1}{N}})d\mu_{1,..,N-1}^I + q^{\frac{1}{N}}d\mu_{1,..,N}^I\Big)$$

where $\mu_{1,..,N-1}^I$ is the probability the prosecutor accepts d in any of the periods from 1 to $N-1$. This probability reflects the fact that the prosecutor best strategy if she gets $y = e$ at period $n = 1$ is offer $x = d$ every period until $n = N-1$. If the innocent defendant rejects $x = d$ at $n = N-1$, the prosecutor discloses $y = e$ at N.

The prosecutor is not going to deviate after period $n = 1$, because if the game has not ended it is because the defendant is innocent. Therefore, investigating is a strictly dominated strategy. The prosecutor is better off deviating if:

$$\theta > \frac{d(\mu_{1,..,N}^I - \mu_{1,..,N-1}^I)}{h - d(1 - \mu_{1,..,N}^I + \mu_{1,..,N-1}^I)} \quad \iff \quad \theta > \frac{d\tilde{\mu}_1^I}{h - d(1 - \tilde{\mu}_1^I)}$$

where $\tilde{\mu}_n^I = \mu_{n,..,N}^I - \mu_{n,..,N-1}^I = (1-\mu_n^I)(1-\mu_{n+1}^I)\cdots(1-\mu_{N-1}^I)\mu_N^I$ is the probability of the innocent defendant accepts $x = d$ at period $n = N$. Then $\frac{d\tilde{\mu}_n^I}{h-d(1-\tilde{\mu}_n^I)} = \underset{\sim}{\theta}^N$, therefore, if $\theta \leq \underset{\sim}{\theta}^N$ the prosecutor is better off no deviating.

Note that deviations after first period are also not profitable, because the prosecutor deviates at n if $\theta^{(n)} > \frac{d\tilde{\mu}_n^I}{h-d(1-\tilde{\mu}_n^I)}$, and note that $\underset{\sim}{\theta}^N < \frac{d\tilde{\mu}_n^I}{h-d(1-\tilde{\mu}_n^I)}$ because $\tilde{\mu}_1^I < \tilde{\mu}_n^I$, therefore the prosecutor does not deviates for $\theta \leq \underset{\sim}{\theta}^N$.

For $\mu_N^I \in [\frac{c}{d+c}, 1]$: If the prosecutor one-shot deviates at period n, her payoff will be:

$$\theta\Big((1-q^{\frac{1}{N}})h + q^{\frac{1}{N}}d\Big) + (1-\theta)\Big((1-q^{\frac{1}{N}})\Big(d\mu_{1,..,N}^I - c(1-\mu_{1,..,N}^I)\Big) + q^{\frac{1}{N}}d\mu_{1,..,N}^I\Big)$$

In this case the prosecutor offers $x = d$ in every period, including period $n = N$, even if she gets $y = e$.

The prosecutor is not going to deviate after period $n = 1$ because if the game has not ended, it is because the defendant is innocent. Therefore, investigating is a strictly dominated strategy.

The prosecutor is better off deviating if:

$$\theta > \frac{(1-\mu_{1,..,N}^I)c}{h-d+(1-\mu_{1,..,N}^I)c}$$

Note $\frac{(1-\mu_{1,..,N}^I)c}{h-d+(1-\mu_{1,..,N}^I)c} = \underset{\sim}{\theta}^N$, therefore, if $\theta \leq \underset{\sim}{\theta}^N$ the prosecutor is better off no deviating.

Lemma 2. The prosecutor's expected payoff for $\theta \leq \underline{\theta}^N$ in the no-investigation equilibrium is

$$u^P = \begin{cases} \theta d + (1-\theta)\mu^I(d)d & \text{if } \theta \in (0, \underset{\sim}{\theta}^N] \\ \theta\Big[(1-q^{\frac{1}{N}})h + q^{\frac{1}{N}}dq^{\frac{1}{N}}\Big] + (1-\theta)dq^{\frac{1}{N}} & \text{if } \theta \in (\underset{\sim}{\theta}^N, \underline{\theta}^N] \end{cases}$$

and under the immediate-agreement is:

$$u^P = \theta\Big[(1-q^{\frac{1}{N}})h + q^{\frac{1}{N}}dq^{\frac{1}{N}}\Big] + (1-\theta)dq^{\frac{1}{N}}$$

For $\mu_N^I \leq \frac{c}{c+d}$: The prosecutor payoff when $\theta \leq \underset{\sim}{\theta}$ is:

$$\theta d + (1-\theta)\mu^I(d)d$$

Note if N increases $\underset{\sim}{\theta}^N$ will change, because there are more probability of accepting $x = d$ μ_n^I. Given $\mu_n^I < 1$ $\forall n \in [0, N]$, the value of $\underset{\sim}{\theta}^N$ is always lower than 1, no matter the number or periods. This implies $\theta d + (1-\theta)\mu^I(d)d < d$. Therefore, for any sequence $s = \{\mu_1^I, \mu_2^I, ...\}$ of extra probabilities of acceptance when N increases, $\lim_{N\to\infty} \theta d + (1-\theta)\mu^I(d)d = d - \upsilon_s$. Therefore, given that:

$$\lim_{N\to\infty} \theta\Big[(1-q^{\frac{1}{N}})h + q^{\frac{1}{N}}dq^{\frac{1}{N}}\Big] + (1-\theta)dq^{\frac{1}{N}} = d$$

Therefore $\exists N^*$ such that $\forall N \geq N^*$, $\theta\Big[(1-q^{\frac{1}{N}})h + q^{\frac{1}{N}} dq^{\frac{1}{N}}\Big] + (1-\theta)dq^{\frac{1}{N}} \geq d - \upsilon_s$.

For $\mu_N^I > \frac{c}{c+d}$: The prosecutor's payoff when $\theta \leq \underset{\sim}{\theta}^N$ is:

$$\theta d + (1-\theta)\mu^I(d)d$$

where $\mu^I(d)$ is the probability of the innocent defendant accepting d at some period between the first one and the last one period. Note $\lim_{N\to\infty} \underset{\sim}{\theta} \leq 1$. If the limit is 1, then $\lim_{N\to\infty} \theta d + (1-\theta)\mu^I(d)d = d$, but $\lim_{N\to\infty} \underset{\sim}{\theta} = 0$. The prosecutor always gets the payoff of immediate agreement.

If $\lim_{N\to\infty} \underset{\sim}{\theta} < 1$, then $\lim_{N\to\infty} \theta d + (1-\theta)\mu^I(d)d = d - \upsilon_S$ and $\lim_{N\to\infty} \underset{\sim}{\theta}^N > 0$. Therefore $\exists N^*$ such that $\forall N \geq N^*$, $\theta\Big[(1-q^{\frac{1}{N}})h + q^{\frac{1}{N}} dq^{\frac{1}{N}}\Big] + (1-\theta)dq^{\frac{1}{N}} \geq d - \upsilon_s$.

Lemma 3. For values of $\theta > \underline{\theta}^{\text{Public}}$, the proof of Lemma 3 is the same than the proof of Proposition 2 and 3. For $\theta < \underline{\theta}^{\text{Public}}$, the prosecutor payoff is $v^P = d$. If the prosecutor deviates at any period $n \in [1, N]$, the defendant is going to observe it and therefore her payoff is going to be:

$$\theta\Big((1-q^{\frac{1}{N}})h + q^{\frac{1}{N}} dq^{\frac{1}{N}}\Big) + (1-\theta)dq^{\frac{1}{N}}$$

The best strategy for the prosecutor is to disclose $y = h$ and to hide $y = e$. On the other hand, the innocent defendant's continuation punishment if there is only one investigation is $q^{\frac{1}{N}}d$.

The prosecutor is better off deviating if: $\theta > \frac{d}{h-dq^{\frac{1}{N}}}$. However $\frac{d}{h-dq^{\frac{1}{N}}} > \frac{d}{h-dq}$, therefore the prosecutor does not deviate.