Negotiation Protocol in Two-Issue Bargaining with Contributions

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Abstract

I study the role of the negotiation protocol in two-issues bargaining between two players, in which the pie only exists if both players contribute to its creation. The issues are the fraction of the pie, and the second is the pie itself, modeled as which project to choose. I examine three protocols—simultaneous, sequential, and incomplete bargaining—and show that the protocol does not play any role if the contribution cost of one player is high enough. I provide conditions under which the protocol plays a role and describe the equilibrium. I apply the model to discuss the allocation of control rights in early projects financed by venture capital.

JEL Codes: C72, C78, D74.

Keywords: Multi-issue bargaining, negotiation protocol, investment.

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1 Introduction

There are two common elements of many bargaining situations: the parties involved negotiate over more than one issue, often with endogenous negotiation protocol, and the parties involved must contribute to generating the surplus they are negotiating over. One application of this is the case of a venture capitalist investing in a startup. She negotiates the shares of the new venture with the entrepreneur who owns the startup, and modifications to the project that she believes will make it more profitable. The entrepreneur might not agree with the venture capitalist or might have private benefits from implementing the project without modifications. In addition, both parties make costly contributions. The venture capitalist contributes the investment, and the entrepreneur contributes the labor.

Regarding the negotiation protocol, the party who acts as proposer can decide whether to negotiate the two issues simultaneously or sequentially, or even negotiate only one issue and leave the second to be determined in the future.

This paper examines a two-issue bargaining model that captures such situations. Two players must contribute to a joint project to generate a surplus, and one of them acts as a proposer and chooses the negotiation protocol. I characterize the equilibrium depending on the negotiation protocol and discuss the importance of being able to select it. Also, I use the model to discuss the role of the control rights—i.e., the right to decide remaining issues in the future—in sequential negotiations.

I show that the negotiation protocol plays a role in the outcome only if the costs of the contributions necessary to generate the surplus are low enough. If the cost of the contribution of one player is high enough, all protocols give the same results in terms of payoff and regarding which project is implemented.

Although I focus on an example of the negotiation between an entrepreneur and a venture capitalist in this paper, the results apply to a more general context.[1] In the model, the venture capitalist and entrepreneur negotiate over which project to implement and its shares. Each player prefers a different project, and each player prefers more shares. The contributions differ in the timing; whereas the investment made by the venture capitalist is agreed on during the negotiation, the entrepreneur decides whether to put effort into the project after the project and its shares are determined.

[1] For example, the central government negotiates with the local government regarding the characteristic of a new public park (dog-friendly, with picnic areas, etc.) and where to locate it. The central government contributes with the funding, and the local government takes care of the park's maintenance.

I consider three possible negotiation protocols that resemble a real-life agenda. The first is simultaneous negotiation, in which the proposer simultaneously makes an offer that consists of a project and the shares for each player. The second one is a sequential negotiation, in which the proposer offers the shares of the project, and if accepted, a project-proposer is randomly elected and offers which project to implement. The third protocol is incomplete bargaining, in which the proposer offers the shares, and if accepted, a decision-maker is randomly elected and decides which project to implement. If the responder rejects the offer in any protocol, the game ends and both players get zero. If the negotiation ends in an agreement, the venture capitalist invests, and then the entrepreneur decides whether to make the costly effort.

The intuition behind why the bargaining protocol does not play a role if the contribution costs are high enough is because, independent of the negotiation protocol, a player with a high contribution cost must be compensated for that contribution to accept any offer. Intuitively, there are two ways that compensation is possible: choosing the player's most preferred project and providing him with a low number of shares or choosing his least preferred project and offering him a large number of shares. If the required compensation is high enough, the former option is better. Given that each player has a different most preferred project—that is, they assign a higher utility to different projects—the *pies* over which they are bargaining are different; nevertheless, the shares are the same. So, offering the player's least preferred project and offering him a large number of shares if the required compensation is high means that the number of shares needs to be close to 100%. Therefore, the proposer gets almost zero percentage of a large pie.

On the other hand, choosing the player's preferred project means that the number of shares given to him is much lower because that player's pie is bigger. Therefore, the proposer gets a more significant number of shares of the smaller pie, which is better than almost no shares of a larger pie.

Lastly, I use the model to discuss the question of when having control rights—and negotiating over them—is optimal. Consider the case in which the negotiation is incomplete. In the first period, players negotiate the shares, and in the second period, someone decides which project to implement. In this case, during the first period the players also negotiate the right to choose the project in the second period. Using the results of the model, I show that having control rights is not helpful if the contribution costs are high enough, because even though a player has the right to choose the project, he will select the preferred project of the player with high contribution costs.

This paper relates to the literature on multi-issue bargaining. A closely related paper is Lang and Rosenthal (2001), in which a proposer can make an offer that includes one or all the issues. They show that restricting offers to include all issues can make the players worse off. My paper differs in that each player must contribute to the surplus, and it also includes the possibility of an incomplete bargaining protocol. Among papers in which the proposer can choose whether to make offers on all issues at once or a subset of them, Busch and Horstmann (1997); and Inderst (2000) present a model with the assumption of separable utility over different issues. In this paper, I focus on the case of non-separable utilities, which generate different conclusions. In and Serrano (2003) study the conditions that induce a unique SPE and show that the offers include all issues at once in that SPE. Fershtman (2000) presents a model in which two members that might have different preferences negotiate against a single player. In the present paper, I assume that the two players bargain against each other. Heifetz and Ponsati (2007) show the benefits of issue-by-issue protocols under asymmetric information compared to the all-at-once protocol. I consider a different setting and complete information.[2]

This paper also relates to the literature on bargaining under disagreement. Van den Steen (2010) presents a model of Nash bargaining in which two parties decide how to allocate the control rights under disagreement. In this paper there is disagreement, but I focus on the effect of the contributions of each player rather than the size of the disagreement.

The question regarding the allocation of control in organizations belongs to a dense literature. In this paper, I focus on allocating control when shares and control are negotiated separately, as described by Kaplan and Strömberg (2004) for venture capital investments. A related paper is that of Hellmann (1998), who presents a model in which the control is the right to appoint a new CEO. He shows that control rights are relinquished by the entrepreneur when the venture capitalist requires it to engage in the costly search for a new CEO, and not only when that is the only option available to obtain the investment. I obtain a similar result on voluntary relinquishing of control, but the reason is that it is less expensive for the entrepreneur to provide control than cash-flow rights.

The paper is organized as follows. Section 2 introduces the model. Section 3 shows the equilibrium for each protocol and compares the result. Section 4 discusses the role of control rights, and Section 5 concludes. All proofs are in the Appendix.

[2]Other papers that study multi-issue bargaining, but with a focus on the order of negotiation are Fershtman (1990); In and Serrano (2004); Flamini (2007); Chen and Eraslan (2017); and Xiao (2018).

2 Model

There are two players: player 1 (he) and player 2 (she). Players bargain over the shares of a project and over which project to implement. Each player has a different preference over projects. I consider that the project belongs to $x \in \{L, R\}$ and that each player associates a value u_i to each project:

$$u_1 = \begin{cases} \bar{v} & \text{if } x = R \\ \underline{v} & \text{if } x = L \end{cases}, \quad \text{and} \quad u_2 = \begin{cases} \underline{v} & \text{if } x = R \\ \bar{v} & \text{if } x = L, \end{cases}$$

with $\bar{v} > \underline{v}$. Player 1 preferred project is $x = R$, and player 2's preferred project is $x = L$. The shares of the project for players 1 and 2 are represented by $(y, 1 - y)$, in which $y \in [0, 1]$.

The assumption that each player associates a different value to each project can be interpreted as either both players openly disagree about the probability of success assigned to each project or have nontransferable private benefits for different projects.[3] Under either assumption, the results remain the same.

Each player has a resource: Player 1 has the effort and player 2 has the investment. Both resources are critical; investment is necessary to start the project, and the effort is necessary for the project's success. The investment corresponds to a value M, and the effort to $e \in \{0, 1\}$ at cost c. The effort is not contractible, and player 1 chooses the effort level after the project and its shares are agreed to and the investment made. If effort $e = 0$ is chosen, the project losses its value and both players get 0.

One of the players is the proposer and has two crucial tasks. First, the proposer chooses the negotiation protocol, and second, the proposer makes the first offer. I consider three negotiation protocols:

1. **Simultaneous negotiation:** The proposer simultaneously offers (x, y). If accepted, player 2 pays the investment and then player 1 chooses the effort level. If the offer is rejected, both players get 0.

2. **Sequential negotiation:** The proposer offers $(y, 1 - y)$. If the offer is rejected, both players get 0;if it is accepted, a project-proposer is elected with probabilities $(q, 1 - q)$ for players 1 and 2, respectively. The project-proposer offers x. If rejected, both players get 0; if accepted, player 2 makes the investment, and then player 1 chooses the effort level.

[3] See Van den Steen (2010) for a detailed discussion of the differing-prior assumption.

3. **Incomplete negotiation:** The proposer offers $(y, 1-y)$. If the offer is rejected, both players get 0; if it is accepted, player 2 invests M. After the investment, a decision-maker is chosen, with probabilities q for player 1 and $1-q$ for player 2. The decision-maker chooses project x, and then player 1 makes the effort decision.

I allow the proposer, project-proposer, and decision-maker to opt out of the negotiation instead of making an offer. If the investment is made, the payoffs are given by the following expressions:

$$U_1 = \begin{cases} yu_1 - c & \text{if } e = 1 \\ 0 & \text{if } e = 0 \end{cases} \quad \text{and} \quad U_2 = \begin{cases} (1-y)u_2 - M & \text{if } e = 1 \\ -M & \text{if } e = 0 \end{cases}$$

An equilibrium is a subgame perfect equilibrium (SPE).

3 Analysis

3.1 Player 1's effort decision

I start by showing the effort decision of player 1, which is made after the negotiation. The effort decision is independent of the negotiation protocol and only depends on the project and the shares.

Player 1 chooses $e = 1$ if the expected return is higher than the cost of the effort. If the project is $x = R$, player 1 makes effort if $y\bar{v} - c \geq 0$. If the project is $x = L$, player 1 makes effort if $y\underline{v} - c \geq 0$. Define $\bar{y} = \frac{c}{\underline{v}}$ and $\underline{y} = \frac{c}{\bar{v}}$.

Lemma 1 *If $y < \underline{y}$, player 1 chooses $e = 0$. If $y \in [\underline{y}, \bar{y})$, player 1 chooses $e = 1$ if $x = R$ and $e = 0$ if $x = R$. If $y \geq \bar{y}$, player 1 chooses $e = 1$.*

Lemma 1 is graphically represented in Figure 1. If shares for player 1 are low enough, player 1 always chooses $e = 0$ because even with his favorite project, the return is too low. If shares are high, he always chooses $e = 1$ because of the high return. If the shares take intermediate values, he only chooses $e = 1$ if his favorite project is implemented.

3.2 Outcome of each protocol

I now focus on the simultaneous and sequential bargaining protocols, and show that the feasible set of (c, M) such that agreement is possible is the same for both protocols.

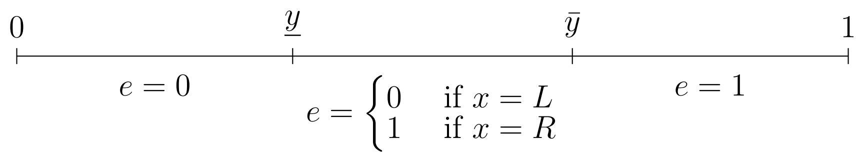


Figure 1: Effort made by player 1 depending on x and y.

After the shares are agreed on, the projects x that are feasible are those that induce player 1 to choose $e = 1$, as stated in Lemma 1, and that provide a return of at least M to player 2. Note that player 1 always accepts any offer, since he can choose $e = 0$ and ensure himself 0, but player 2 must get a return of M to accept the offer, if she is the responder, or not opt out if she is the proposer of x. Therefore, in both protocols, to be feasible for the proposer to offer $x = L$, there is a y such that player 2 is willing to accept it if

$$(1 - \bar{y})\bar{v} - M \geq 0 \iff 1 \geq \frac{M}{\bar{v}} + \frac{c}{\underline{v}}, \tag{1}$$

and to be feasible for the proposer of the project to offer $x = R$, there is a y such that player 2 is willing to accept it if

$$(1 - \underline{y})\underline{v} - M \geq 0 \iff 1 \geq \frac{M}{\underline{v}} + \frac{c}{\bar{v}}. \tag{2}$$

Equations (1) and (2) determine the combination of (c, M) such that an agreement is feasible, as represented in Figure 2.

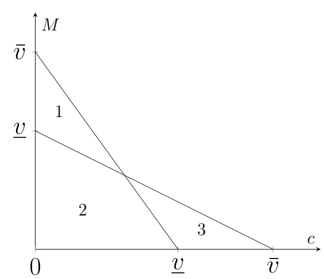


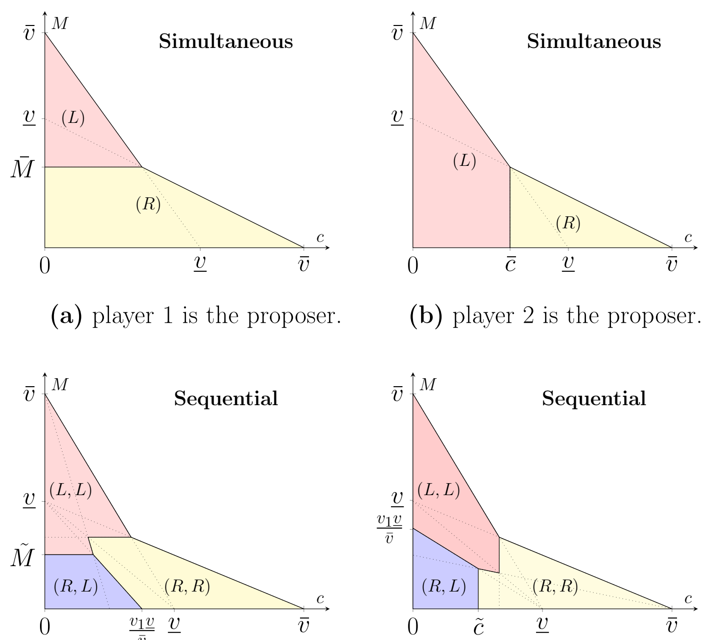
Figure 2: Combinations of c and M in which an agreement is possible.

In area 1 of Figure 2, only $x = L$ is feasible because choosing $x = R$ requires allocating a significant portion of the shares to player 2 to compensate her for her investment M in a project with a lower expected return. In this case, the remaining shares for player 1 are not enough to induce him to make effort. Symmetrically, in area, 3 only $x = R$ is feasible because if the project is $x = L$, it would require a big portion of shares for player 1 to compensate him for the cost of his effort, and it makes it not profitable for player 2 to invest. Both $x = R$ and $x = L$ are feasible if (c, M) belongs to area 2. Consider the constants $\bar{c}$ and $\bar{M}$ and the functions $\bar{c}(M)$ and $\bar{M}(c)$ defined in Appendix A.2.

Proposition 1 *For simultaneous and sequential bargaining protocols, if player 1 is the proposer the equilibrium outcome is $x = L$ and $y = 1 - \frac{M}{\bar{v}}$ if $M \geq \bar{M}$, and $x = R$ and $y = 1 - \frac{M}{\underline{v}}$ if $c \geq \bar{c}(M)$. If player 2 is the proposer the equilibrium outcome is $x = L$ and $y = \frac{c}{\underline{v}}$ if $M \geq \bar{M}(c)$, and $x = R$ and $y = \frac{c}{\bar{v}}$ if $c \geq \bar{c}$.*

Proposition 1 shows that if c or M are high enough, the bargaining protocol does not determine which project is implemented in equilibrium. It also shows that the proposer is indifferent between them, since both provide the same payoff. For smaller values of (c, M), the equilibrium depends on who is the proposer and the specific values of the cost.

complete characterization of the equilibrium is in Appendix A.2. Informally, the equilibrium for both protocols is that the proposer offers their preferred project if the contribution cost of the other player is not high enough. Figure 3 shows the representation of the equilibrium.



(a) player 1 is the proposer. **(b)** player 2 is the proposer.

(c) player 1 is the proposer. **(d)** player 2 is the proposer.

Figure 3: Projects implemented in equilibrium for simultaneous and sequential bargaining.

The proposer's trade-off drives the above results. For the simultaneous protocol, the decision can be interpreted as the player who is the proposer choosing between two fractions of two different pies. Suppose player 1 is the proposer:

- **Small fraction of a big pie:** If player 1 offers $x = R$, he must provide significant shares to player 2 to induce her to accept it. Therefore, from his point of view, he gets a return equal to a small fraction of the big pie $\bar{v}$.

- **Large fraction of a small pie:** If the offer is $x = L$, player 1 provides a small amount of shares to player 2 to induce her to accept it. In that case, he gets a more significant fraction of a smaller pie $\underline{v}$.

Note that player 2 gets the exact opposite pie. If player 1 gets a fraction of the smaller pie, player 2 receives a fraction of the big pie.

If M is small, it represents a small fraction of both pies for player 2: the big ($x = L$) and the small ($x = R$). Thus player 1 prefers to choose his big pie ($x = R$), since he receives a significant fraction of it. However, as M increases, the fraction it represents of $\underline{v}$ increases much faster than the fraction it represents of $\bar{v}$. Therefore, player 1 prefers to choose $x = L$ if M is large enough. He gets a more significant fraction of $\underline{v}$, because otherwise M is a substantial fraction of $\underline{v}$ and this implies that player 1 gets a tiny fraction of $\bar{v}$. The analysis if player 2 is the proposer is symmetric.[4]

For sequential bargaining, in addition to the option for the proposer of choosing a small fraction of a big pie and a large fraction of a small pie, there is also the option of choosing an intermediate number of shares of a medium pie (represented by the case in which a player chooses their preferred project if elected as the proposer of x). The last option is optimal for small (c, M) for two reasons: (1) getting a big fraction of the small pie is only optimal if M is high, as discussed in the simultaneous bargaining, and (2) to induce $x = R$, the shares y that player 1 must offer to player 2, in this case, are fewer than if inducing R in the simultaneous bargaining. That is, player 1 gets even fewer shares of the big pie.

I now consider the incomplete bargaining protocol. In this case, the feasible combinations of (c, M) that induce an equilibrium are lower. The reason is that the decision-maker chooses x after the investment is made, implying that player 1 always chooses $x = R$. In this case, player 2 accepts an offer that, in expectation, gives her a return of M. This means that if player 1 is elected decision-maker, player 2 might have a

[4]Furthermore, given that both players incur in the contribution costs only after the decisions of (x, y) are made, both players are ex ante symmetric.

return lower than M under that realization of the decision-maker. This differs from the sequential protocol, in which player 2 must get a return of at least M under each realization of the project-proposer. Therefore, to be feasible for player 2 to choose $x = L$ if she is the decision-maker, there is a y such that player 2 is willing to accept if

$$(1-\bar{y})(q\underline{v}+(1-q)\bar{v}) - M \geq 0 \iff 1 \geq \frac{M}{v_2} + \frac{c}{\underline{v}}, \tag{3}$$

in which $v_2 = q\underline{v} + (1-q)\bar{v}$. The combinations of (c, M) such that an agreement is feasible are given by equations (2) and (3), as shown in Figure 4.

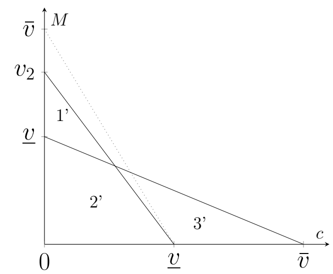
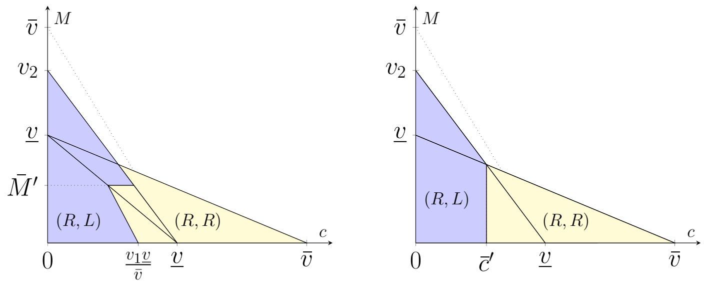


Figure 4: Combinations of c and M in which an agreement is possible.

Similar to the other protocols, in area $3'$, only $x = R$ is feasible because if the project is $x = L$, it would require a big portion of shares for player 1 to compensate him for the cost of the effort, and this makes it not profitable for player 2 to invest. The difference is in area $1'$, in which the only option is that player 2 chooses $x = L$ and player 1 chooses $x = R$. In area $2'$, an equilibrium in which player 2 chooses $x = L$ or $x = R$ is feasible. Consider the constant $\bar{c}'$ and a function $\bar{c}(M)'$ defined in Appendix A.3.

Proposition 2 *For incomplete bargaining, if player 1 is the proposer the equilibrium outcome is $x_1 = R$, $x_2 = L$, and $y = 1 - \frac{M}{\underline{v}}$ if $c \leq \bar{c}'$, and $x = R$ otherwise. If player 2 is the proposer the equilibrium outcome is $x_1 = R$, $x_2 = L$, and $y = \frac{c}{\underline{v}}$ if $c \geq \bar{c}(M)'$ and $x = R$ otherwise.*

Proposition 2 shows that if c is high enough, the project implemented is $x = R$ no matter who is the proposer. If c is smaller, the project implemented is $x = R$ if player 1 is elected as the decision-maker and $x = L$ if player 2 is the decision-maker. The full description of the equilibrium is in Appendix A.3. Figure 5 graphically shows the project-selection equilibrium.



(a) player 1 is the proposer. **(b)** player 2 is the proposer.

Figure 5: Project implemented in equilibrium for incomplete bargaining.

3.3 Equilibrium

I now present the main results of the paper.

Proposition 3 *Player 1 is indifferent between simultaneous and sequential protocols if $M \geq \bar{M}$; indifferent between simultaneous, sequential, and incomplete protocols if $c \geq \bar{c}(M)$.*

Player 2 is indifferent between simultaneous and sequential protocols if $M \geq \bar{M}(c)$, indifferent between simultaneous, sequential, and incomplete protocols if $c \geq \bar{c}$.

Proposition 3 shows that if player 1 is the proposer and M is high, he is indifferent between the simultaneous and sequential bargaining protocols. Note that in incomplete bargaining, he can implement $x = R$ with probability q, but he prefers to implement $x = L$ for sure. By choosing the incomplete bargaining protocol, he must allocate more shares to player 2 than by choosing either the simultaneous or sequential protocol. The effect of a smaller pie but higher shares dominates over fewer shares but a larger pie, because a high M implies that the relinquished shares must be significant. Therefore, player 1 prefers to choose $x = L$.

Player 1 is indifferent among the three protocols if c is high, because the equilibrium project is $x = R$. For player 2 the incomplete bargaining protocol is a dominated option, because once the investment is made, there is no way for player 2 to induce player 1 to choose $x = L$. Therefore, forgoing the option of choosing the project is never optimal.

Corollary 1 *The project implemented is $x = R$ if $c \geq \bar{c}$, and $x = L$ if $M \geq \bar{M}$.*

Corollary 1 says that independent of who is the proposer, in equilibrium, player 1's preferred project is implemented if his cost of effort is high, and player 2's preferred project is implemented if her investment costs are high. As the cost gets larger, the proposer must allocate a very large amount of shares if choosing the non-preferred project of the player with the high cost, which is not optimal. It is better for the proposer to allocate fewer shares of the preferred project of the player with high contribution cost.

4 Discussion: The role of control rights

In venture capitalist contracts, a natural question is which party (the venture capitalist or the entrepreneur) holds the control rights of the organization, i.e., the formal rights to decide on issues in the future. Some of the most common decisions are the right to appoint a new CEO, financial choices, and project decisions.

In this section, I use the model to answer the question of under what conditions an entrepreneur is willing to relinquish control rights—the formal authority to make decisions inside the organization—to the venture capitalist instead of keeping them for himself, in the context of choosing the project. I consider that a venture capitalist and entrepreneur negotiate the investment in a project, the shares, and the control rights. Control rights are the right to choose x.

I make the following two modifications to the model: (1) I consider the structure of the incomplete bargaining model, but the proposer's offer is (y, q). The proposer's offer is the shares of the project and the value of q. (2) The values that q can take are $q \in \{0, 1\}$. The proposer allocates control rights to him/herself or the other player.

Corollary 2 *If the entrepreneur (player 1) is the proposer, he relinquishes control rights if M is large enough and c is low. If the venture capitalist (player 2) is the proposer, she offers control rights to the entrepreneur ($q = 1$) if c is large enough.*

This result comes directly from replacing $q = 1$ and $q = 0$ in Proposition 2. Corollary 1 says that both players prefer to allocate control rights to the other player when the costs of the other's player contribution are high enough. Three features of the result are worth highlighting:

1. For large c, independent of who is the proposer, control rights are irrelevant because both players will choose $x = R$ to induce the entrepreneur to make effort $e = 1$. In the sense of Aghion and Tirole (1997), the relevant control is given by the *real authority* rather the *formal control.*

2. As in Hellmann (1998), for some combinations of M, the entrepreneur does not relinquish control rights because it is the only way to secure the investment; in-

stead, it is optimal for him/her to do it. Keeping the control rights and providing more cash flow rights is feasible, but not optimal.

3. This model explains the allocation of control rights without assuming the private benefit of having control rights. Rather, it focuses on what is possible to do with the control rights.

5 Final Remarks

The paper shows that in a two-issues bargaining model, in which each player makes a costly contribution, the negotiation protocol does not play any role if the cost of a contribution is high enough. In that case, the identity of the proposer does not matter for project selection.

I close by discussing the role of the parameter q. The incomplete bargaining protocol is inefficient if $q > 0$, because the region (c, M) is lower than the simultaneous and sequential bargaining protocols. The expected return player 2 must get to accept an agreement must be at least M, but if player 1 is elected as decision-maker, he chooses $x = R$ no matter the shares. In turn, player 2 gets a lower return than M if player 1 is the decision-maker, instead of sequential bargaining, in which player 2 gets at least M for each realization of the project-proposer. As the probability of player 2's being the decision-maker increases, the incomplete bargaining protocol increases its efficiency.

A Appendix

A.1 Proof of Lemma 1:

If the project is $x = R$, player 1 makes effort if the shares of the projects that he gets are such that $y\bar{v} - c \geq 0$, and if the project is $x = L$, player 1 makes effort if the shares are such that $y\underline{v} - c \geq 0$.

Define $\bar{y} = \frac{c}{\underline{v}}$ and $\underline{y} = \frac{c}{\bar{v}}$. If $y < \underline{y}$, player 1 chooses $e = 0$ because $\bar{v}\underline{y} - c = 0 > \bar{v}y - c > \underline{v}y - c$. That is, with $x = L$ or $x = R$ player 1 gets a negative payoff if $e = 1$. If $y \in [\underline{y}, \bar{y})$ player 1 chooses $e = 1$ if $x = R$ and $e = 0$ if $x = L$ because $y\bar{v} - c \geq \underline{y}\bar{v} - c = 0$, and $0 = \bar{y}\underline{v} - c > y\underline{v} - c$. If $y \geq \bar{y}$ player 1 chooses $e = 1$ no matter the project because $y\bar{v} - c > y\underline{v} - c \geq \bar{y}\underline{v} - c = 0$.

A.2 Proof of Proposition 1

For simultaneous bargaining protocol:

Election of x. Suppose player 1 offer the contract. The highest y he can get is:

- $y = 1 - \frac{M}{\underline{v}}$ if $x = R$ because $(1-y)\underline{v} - M \geq 0$,
- or $y = 1 - \frac{M}{\bar{v}}$ if $x = L$ because $(1-y)\bar{v} - M \geq 0$.

Player 1 offers $x = R$ if $\left(1 - \frac{M}{\underline{v}}\right)\bar{v} - c \geq \left(1 - \frac{M}{\bar{v}}\right)\underline{v} - c \iff M \leq \frac{\bar{v}\underline{v}}{\bar{v}+\underline{v}} \equiv \bar{M}$.

Suppose player 2 offers the contract. The highest $(1-y)$ she can get is:

- $y = \frac{c}{\underline{v}}$ if $x = L$ because $y\underline{v} - c \geq 0$,
- or $y = \frac{c}{\bar{v}}$ if $x = R$ because $y\bar{v} - c \geq 0$.

Player 2 offers $x = L$ if $\left(1 - \frac{c}{\underline{v}}\right)\bar{v} - M \geq \left(1 - \frac{c}{\bar{v}}\right)\underline{v} - M \iff c \leq \frac{\bar{v}\underline{v}}{\bar{v}+\underline{v}} \equiv \bar{c}$.

Feasibility of election of x. Suppose player 1 chooses $x = R$ (provided $M \leq \bar{M}$). It is feasible for player 1 to choose $x = R$ if $\left(1 - \frac{M}{\underline{v}}\right)\bar{v} - c \geq 0 \iff 1 \geq \frac{c}{\bar{v}} + \frac{M}{\underline{v}}$. Similarly, it is feasible for player 1 to choose $x = L$ if $\left(1 - \frac{M}{\bar{v}}\right)\underline{v} - c \geq 0 \iff 1 \geq \frac{c}{\underline{v}} + \frac{M}{\bar{v}}$.

Suppose player 2 choose $x = L$ (provided $c \leq \bar{c}$). It is feasible for player 2 to choose $x = L$ if $\left(1 - \frac{c}{\underline{v}}\right)\bar{v} - M \geq 0 \iff 1 \geq \frac{c}{\underline{v}} + \frac{M}{\bar{v}}$. Similarly, it is feasible for player 2 to choose $x = R$ if $\left(1 - \frac{c}{\bar{v}}\right)\underline{v} - M \geq 0 \iff 1 \geq \frac{c}{\bar{v}} + \frac{M}{\underline{v}}$.

In Figure 2 (Section 3.2), the feasible combinations are:

- Area 1: only L is feasible
- Area 2: both R and L are feasible
- Area 3: only R is feasible

The decision if player 1 is the proposer are:

- $(y, x) = (1 - \frac{M}{\underline{v}}, R)$ if $M \leq \bar{M}$. $U_1 = (1 - \frac{M}{\underline{v}})\bar{v} - c$, $U_2 = \frac{M}{\underline{v}}\underline{v} - M = 0$
- $(y, x) = (1 - \frac{M}{\bar{v}}, L)$ if $M > \bar{M}$. $U_1 = (1 - \frac{M}{\bar{v}})\underline{v} - c$, $U_2 = \frac{M}{\bar{v}}\bar{v} - M = 0$

The decision if player 2 is the proposer are:

- $(y, x) = (\frac{c}{\underline{v}}, L)$ if $c \leq \bar{c}$. $U_2 = (1 - \frac{c}{\underline{v}})\bar{v} - M$, $U_1 = \frac{c}{\underline{v}}\underline{v} - c = 0$
- $(y, x) = (\frac{c}{\bar{v}}, R)$ if $c > \bar{c}$. $U_2 = (1 - \frac{c}{\bar{v}})\underline{v} - M$, $U_1 = \frac{c}{\bar{v}}\bar{v} - c = 0$

For sequential bargaining protocol:

Election of x. For a fixed y, if player 1 is elected project-proposer, he chooses $x = R$ if $y \leq 1 - \frac{M}{\underline{v}}$ and $x = L$ if $y \in (1 - \frac{M}{\underline{v}}, 1 - \frac{M}{\bar{v}}]$, otherwise player 2 decides not to invest. Call $\bar{y}_2 = 1 - \frac{M}{\bar{v}}$ and $\underline{y}_2 = 1 - \frac{M}{\underline{v}}$.

If player 2 is elected project-proposer, she chooses $x = L$ if $y \geq \frac{c}{\underline{v}}$ and $x = R$ if $y \in [\frac{c}{\bar{v}}, \frac{c}{\underline{v}})$, otherwise player 2 decides not to invest. Call $\bar{y}_1 = \frac{c}{\underline{v}}$ and $\underline{y}_1 = \frac{c}{\bar{v}}$.

Feasibility of x if player 1 is the project-proposer. For $x = R$, player 1 chooses $e = 1$ if $y \geq \frac{c}{\bar{v}}$, while if $x = L$, player 1 chooses $e = 1$ if $y \geq \frac{c}{\underline{v}}$. Therefore, choosing $x = R$ is feasible if $y \in [\frac{c}{\bar{v}}, 1 - \frac{M}{\underline{v}}] \iff 1 \leq \frac{c}{\bar{v}} + \frac{M}{\underline{v}}$, and choosing $x = L$ is feasible if $y \in [\frac{c}{\underline{v}}, 1 - \frac{M}{\bar{v}}] \iff 1 \leq \frac{c}{\underline{v}} + \frac{M}{\bar{v}}$. See Figure 6.

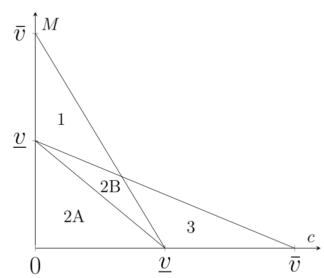


Figure 6: Combinations of c and M in which an agreement is possible.

For area 1 in Figure 6 the only feasible option is to choose $x = L$. If player 1 offers the shares, he offers $y = \bar{y}_2$, which provides a payoff of $\bar{y}_2 \underline{v} - c \geq 0$. Similarly, in area 3 in Figure 6, the only feasible option is to choose $x = R$. Player 1 chooses $y = \underline{y}_2$.

In area 2 of Figure 6, player 1 can choose y such that the elections of x are $(x_1, x_2) = \{(R, L), (R, R), (L, L)\}$. (R, L) is feasible if $\underline{y}_2 \geq \bar{y}_1 \iff \underline{v} \geq M + c$ (area 2A), in which case player 1 chooses $y = \underline{y}_2$. (R, R) is feasible if $\underline{y}_2 \geq \underline{y}_1$ and $\underline{y}_2 \leq \bar{y}_1$ which is feasible if $\underline{v} \geq M + c$ and $1 \geq \frac{M}{\underline{v}} + \frac{c}{\bar{c}}$ (area 2B), and in which case player 1 chooses $\underline{y}_2$. (R, R) is also feasible if $\underline{y}_2 \geq \bar{y}_1 \iff \underline{v} \geq M + c$ (area 2A) in which case player 1 chooses $\bar{y}_1$. Lastly, (L, L) is feasible if $\bar{y}_2 \geq \bar{y}_1 \iff 1 \geq \frac{M}{\bar{v}} + \frac{c}{\underline{v}}$ (area 2), in which case $y = \bar{y}_2$.

Feasibility:

- Area 1: only (L, L) is feasible
- Area 2A: (R, L), (R, R) and (L, L) are feasible
- Area 2B: (R, R) and (L, L) are feasible

- Area 3: only (R,R) is feasible

Election of y if player 1 is the proposer. Considering area 2A, $u_1(R,L) \geq u_1(R,R) \iff v_1 \geq c\frac{\bar{v}}{\underline{v}} + M\frac{v_1}{\underline{v}}$, $u_1(R,R) \geq u_1(L,L) \iff \underline{v} < c\frac{\bar{v}}{\underline{v}} + M\frac{\underline{v}}{\bar{v}}$, and $u_1(R,L) \geq u_1(L,L) \iff M \leq (v_1 - \underline{v})\frac{\bar{v}\underline{v}}{v_1\bar{v}-\underline{v}^2} = \frac{q\bar{v}\underline{v}}{q\bar{v}+\underline{v}} \equiv \tilde{M}$. Considering area 2B, $u_1(R,R) \geq u_1(L,L) \iff M \leq \bar{M}$. Figure 3 panel (C) shows the optimal decisions for player 1.

If player 1 is the proposer, the optimal decision is such that:

- To induce (R,R) in area 2A, player 1 chooses $y = \bar{y}_1$. Payoffs are: $U_1 = \frac{c}{\underline{v}}\bar{v} - c = c(\frac{\bar{v}-\underline{v}}{\underline{v}})$ and $U_2 = (1 - \frac{c}{\underline{v}})\underline{v} - M = \underline{v} - c - M \geq 0$
- To induce (R,R) in areas 2B and 3, player 1 chooses $y = \underline{y}_2$. Payoffs are: $U_1 = (1 - \frac{M}{\underline{v}})\bar{v} - c$ and $U_2 = \frac{M}{\underline{v}}\underline{v} - M = 0$
- To induce (R,L) player 1 chooses $y = \underline{y}_2$. Payoffs are: $U_1 = (1 - \frac{M}{\underline{v}})v_1 - c$ and $U_2 = \frac{M}{\underline{v}}v_2 - M = M(1-q)(\frac{\bar{v}-\underline{v}}{\underline{v}}) \geq 0$
- To induce (L,L) player 1 chooses $y = \bar{y}_2$. Payoffs are: $U_1 = (1 - \frac{M}{\bar{v}})\underline{v} - c$ and $U_2 = \frac{M}{\bar{v}}\bar{v} - M = 0$

Feasibility of x if player 2 is the proposer. For the optimal decision of player 2, in area 1 the only feasible decision is $x = L$, and she chooses $\underline{y}_2$ which provides her a profit of $\underline{y}_2\bar{v} - M \geq 0$. Similarly, in area 3 in Figure 6, the only feasible option is to choose $x = R$. Player 2 chooses $y = \underline{y}_1$.

In area 2 of Figure 6, player 2 can choose y such that the elections of x are $(x_2, x_1) = \{(L,R),(L,L),(R,R)\}$. (L,R) is feasible if $\underline{y}_2 \geq \bar{y}_1 \iff \underline{v} \geq M + c$ (area 2A), in which case player 2 chooses $y = \bar{y}_1$. (L,L) is feasible if $\bar{y}_1 \geq \underline{y}_2$ and $\bar{y}_2 \geq \bar{y}_1$ which is feasible if $\underline{v} \geq M + c$ and $1 \geq \frac{M}{\bar{v}} + \frac{c}{\underline{c}}$ (area 2B), and in which case player 2 chooses $\bar{y}_1$. (L,L) is also feasible if $\underline{y}_2 \geq \bar{y}_1 \iff \underline{v} \geq M + c$ (area 2A) in which case player 2 chooses $\underline{y}_2$. Lastly, (R,R) is feasible if $\underline{y}_2 \geq \underline{y}_1 \iff 1 \geq \frac{M}{\underline{v}} + \frac{c}{\bar{v}}$ (area 2), in which case $y = \underline{y}_1$.

Election of y if player 2 is the proposer. Considering area 2A, $u_2(R,L) \geq u_2(L,L) \iff v_2 \geq M\frac{\bar{v}}{\underline{v}} + c\frac{v_2}{\underline{v}}$, $u_2(L,L) \geq u_2(R,R) \iff \underline{v} < M\frac{\bar{v}}{\underline{v}} + c\frac{\underline{v}}{\bar{v}}$, and $u_2(R,L) \geq u_2(R,R) \iff c \leq (v_2 - \underline{v})\frac{\bar{v}\underline{v}}{v_2\bar{v}-\underline{v}^2} = \frac{(1-q)\bar{v}\underline{v}}{(1-q)\bar{v}+\underline{v}} \equiv \tilde{c}$. Considering area 2B, $u_2(L,L) \geq u_2(R,R) \iff c \leq \bar{c}$. Figure 3, panel (d) shows the optimal decisions for player 2.

If player 2 is the proposer, the optimal decision is such that:

- To induce (L, L) in area 2A, player 2 chooses $y = \bar{y}_2$. Payoffs are: $U_2 = \frac{M}{\underline{v}}\bar{v} - M = M(\frac{\bar{v}-\underline{v}}{\underline{v}})$ and $U_1 = (1 - \frac{M}{\underline{v}})\underline{v} - c = \underline{v} - c - M \geq 0$
- To induce (L, L) in areas 2B and 1, player 2 chooses $y = \bar{y}_1$. Payoffs are: $U_2 = (1 - \frac{c}{\underline{v}})\bar{v} - M$ and $U_1 = \frac{c}{\underline{v}}\underline{v} - c = 0$
- To induce (R, L) player 2 chooses $y = \bar{y}_1$. Payoffs are: $U_2 = (1 - \frac{c}{\underline{v}})v_2 - M$ and $U_1 = \frac{c}{\underline{v}}v_1 - c = cq(\frac{\bar{v}-\underline{v}}{\underline{v}}) \geq 0$
- To induce (R, R) player 2 chooses $y = \underline{y}_1$. Payoffs are: $U_2 = (1 - \frac{c}{\bar{v}})\underline{v} - M$ and $U_1 = \frac{c}{\bar{v}}\bar{v} - c = 0$

A.3 Proof of Proposition 2

Election of x. A decision-maker is allocated after y is decided and M paid. player 1 is elected decision-maker with probability q and player 2 with probability $(1 - q)$. In this case player 1 always chooses $x = R$. However, player 2 takes into account the effort decision of player 1.

For a given y, for $x = L$ player 1 chooses $e = 1$ if $y \geq \frac{c}{\underline{v}} \equiv \bar{y}$, and for $x = R$ player 1 chooses $e = 1$ if $y \geq \frac{c}{\bar{v}} \equiv \underline{y}$. Therefore, if $y \geq \bar{y}$ player 2 chooses $x = L$, if $y \in [\underline{y}, \bar{y})$ player 2 chooses $x = R$, and if $y < \underline{y}$ player 2 chooses any x because player 1 chooses $e = 0$.

Election of y by player 2. Define $v_2 = q\underline{v} + (1 - q)\bar{v}$. If player 2 chooses she has basically two options. First option is to choose $y = \bar{y}$, that implies that if she is elected a proposer of x she chooses $x = L$ which provides her a payoff of $(1 - \bar{y})v_2 - M$, or to choose $y = \underline{y}$ that implies that if she is elected a proposer of x she chooses $x = R$ which provides her a payoff of $(1 - \underline{y})\underline{v} - M$.

Choosing y such that decisions are $(x_1, x_2) = (R, L)$ is feasible if $1 - \frac{M}{v_2} \geq \bar{y} \iff 1 \geq \frac{c}{\underline{v}} + \frac{M}{v_2}$. Similarly, choosing y such that (R, R) is feasible if $1 - \frac{M}{\underline{v}} \geq \underline{y}$. Figure 7 shows the different combinations of (M, c) under which the election of y is feasible.

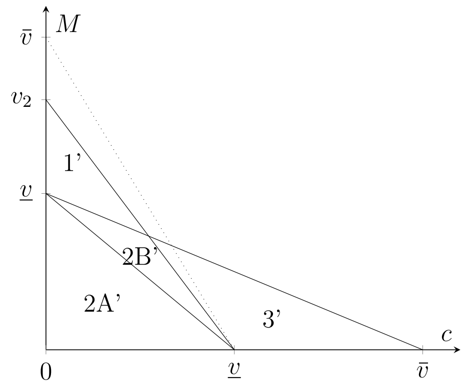


Figure 7: Combinations of c and M in which an agreement is possible.

Feasibility:

- Area 1': only (R, L) is feasible
- Area 2': both (R, L) and (R, R) are feasible
- Area 3': only (R, R) is feasible

In area 2' of Figure 7, player 2 chooses y such that (R, L) if $(1-\bar{y})v_2 - M \geq (1-\underline{y})\underline{v} - M \iff c \leq (v_2 - \underline{v})\frac{\bar{v}\underline{v}}{v_2\bar{v}-\underline{v}^2} = (1-q)\frac{\bar{v}\underline{v}}{(1-q)\bar{v}+\underline{v}} \equiv \bar{c}'$. Optimal decision is represented in Figure 5, panel (b). The decision if player 2 is the proposer are:

- $y = \bar{y}$ if $c \leq \bar{c}'$ which induces (R, L). $U_2 = (1-\bar{y})v_2 - M$, $U_1 = \bar{y}v_1 - c = cq\frac{(\bar{v}-\underline{v})}{\underline{v}} \geq 0$
- $y = \underline{y}$ if $c > \bar{c}'$ which induces (R, R). $U_2 = (1-\underline{y})\underline{v} - M$, $U_1 = 0$

Note that player 1 has a positive payoff if $c \leq \bar{c}'$ and $q > 0$.

Election of y by player 1. Similarly to the election of player 2, player 1 has two options: to induce $(x_1, x_2) = (R, L)$ or (R, R), in which case he chooses $y = 1 - \frac{M}{v_2}$ and $y = 1 - \frac{M}{\underline{v}}$ respectively. Considering $y = 1 - \frac{M}{v_2}$, it is feasible (player 2 is willing to choose R) if $1 - \frac{M}{v_2} \geq \frac{c}{\underline{v}} \iff 1 \geq \frac{M}{v_2} + \frac{c}{\underline{v}}$. Also, player 1 is willing to offer $y = 1 - \frac{M}{v_2}$ if $(1 - \frac{M}{v_2})v_1 - c \geq 0 \iff 1 \geq \frac{c}{v_1} + \frac{M}{v_2}$.

If player 1 wants to induce (R, R), he offers $y = \min\{1 - \frac{M}{\underline{v}}, \bar{y}\}$ so player 2 chooses $x = R$ if $\underline{y} \leq 1 - \frac{M}{\underline{v}} \iff 1 \geq \frac{c}{\bar{v}} + \frac{M}{\underline{v}}$.

Note that to induce (R, R), player chooses $y = \bar{y}$ if $\bar{y} < 1 - \frac{M}{\underline{v}} \iff c + M < \underline{v}$, that is, if (c, M) belongs to area 2A'. In area area 2A' player 1 induces (R, R) instead of (R, L) if $\frac{c}{\underline{v}}\bar{v} - c \geq (1 - \frac{M}{v_2})v_1 - c \iff v_1 \leq c\frac{\bar{v}}{\underline{v}} + M\frac{v_1}{v_2}$.

In area 2B' of Figure 7, player 1 chooses y such that (R, L) if $(1 - \frac{M}{v_2})v_1 - c \geq (1 - \frac{M}{\underline{v}})\bar{v} - c \iff M \geq \frac{\underline{v}v_2}{\bar{v}+\underline{v}} \equiv \tilde{M}$. Optimal decision is represented in Figure 5, panel (a). The decision if player 1 is the proposer are:

- If inducing (R, L) as shown in Figure 5, panel (a), player 1 chooses $y = (1 - \frac{M}{v_2})$. $U_1 = (1 - \frac{M}{v_2})v_1 - c$, $U_2 = \frac{M}{v_2}v_2 - M = 0$.
- If inducing (R, R) in areas 2B' and 3 as shown in Figure 5, panel (a), player 1 chooses $y = (1 - \frac{M}{\underline{v}})$. $U_1 = (1 - \frac{M}{\underline{v}})\bar{v} - c$, $U_2 = \frac{M}{\underline{v}}\bar{v} - M = 0$.
- If inducing (R, R) in area 2A' as shown in Figure 5, panel (a), player 1 chooses $y = \frac{c}{\underline{v}}$. $U_1 = \frac{c}{\underline{v}}\bar{v} - c$, $U_2 = (1 - \frac{c}{\underline{v}})\underline{v} - M = \underline{v} - c - M \geq 0$.

A.4 Proof of Proposition 3

Define:

$$\bar{c}(M)' = \begin{cases} \underline{v} - M & \text{if } M \leq \bar{M}' \\ \underline{v}(1 - \frac{M}{v_2}) & \text{if } M \in (\bar{M}', \bar{M}] \\ \infty & \text{if } M > \bar{M} \end{cases} \quad \text{and} \quad \bar{M}(c)' = \begin{cases} \underline{v} - c & \text{if } c \leq \bar{c} \\ \underline{v} - c\frac{\bar{v}}{v_1} & \text{if } c \in (\bar{c}, \tilde{c}] \\ \bar{v} - c\left(\frac{\bar{v}}{\underline{v}}\right)^2 & \text{if } c \in (\tilde{c}, \bar{c}] \\ \infty & \text{if } c > \bar{c} \end{cases}$$

For player 1 if $M \geq \bar{M}$, simultaneous and sequential bargaining provides player 1 with $U_1 = \left(1 - \frac{M}{\bar{v}}\right)\underline{v} - c$, while incomplete bargaining provides player 1 $U_1 = \left(1 - \frac{M}{v_2}\right)v_1 - c$. If $c \geq \bar{c}(M)$, the three protocols provides him $U_1 = \left(1 - \frac{M}{\underline{v}}\right)\bar{v} - c$.

For player 2 if $M \geq \bar{M}(c)$, simultaneous and sequential bargaining provides player 2 with $U_2 = \left(1 - \frac{c}{\underline{v}}\right)\bar{v} - M$, while incomplete bargaining provides player 1 $U_2 = \left(1 - \frac{c}{\underline{v}}\right)v_2 - M$. If $c \geq \bar{c}(M)$, the three protocols provides him $U_2 = \left(1 - \frac{c}{\bar{v}}\right)\underline{v} - M$.

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